

EFFECTS OF USING THE JAPANESE ABACUS METHOD UPON THE ADDITION AND MULTIPLICATION PERFORMANCE OF GRADE 3 INDONESIAN STUDENTS

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Abstract

This quasi-experimental study was conducted to determine the effect on accuracy and finish time of using the Japanese abacus method on the addition and multiplication performance of thirty Grade 3 pupils in selected schools in Indonesia. Fifteen formed the experimental group, were enrolled in abacus training classes outside of their respective schools, and were taught personally by the researcher through additional abacus treatment for ten meetings. The rest of the students formed the control group who do not use the abacus. Both groups were given the same pretest and posttest on addition and multiplication based on the Indonesian curriculum. Afterwards, students' scores and finish time were analyzed using F-test and Student's t-test. Results show that there is a significant difference between the experimental and control groups in terms of finish time in addition, scores in multiplication, and finish time in multiplication. Results also show that there is a significant increase in multiplication score in the experimental group from pretest to posttest. Student interviews, observations, and analyses of sample solutions revealed several errors that were parallel to Stigler's classification.

Keywords: addition and multiplication performance, Japanese abacus method, mental abacus

Introduction

Doing computation, whether manually or mentally, is a basic component in the process of learning mathematics. Similarly, students need to learn the basic operations in mathematics such as addition, subtraction, multiplication and division, before proceeding to more complicated computational tasks. Westcott and Smith (1968) mentioned that understanding of upper-level concepts relies on the mastery of earlier concepts. Moreover, teachers, parents, tutors, and students themselves have a tendency to search for a technique or method that can be used to more easily teach or learn basic mathematical skills – ideally, to lead to mastery. There are plenty of methods and tools that can be utilized to teach speedy and accurate computation; one of which is by using the abacus – an ancient

calculating device used primarily in Asian culture for performing arithmetic processes (Gera and Kaur, 2014).

Motivated by the situation encountered in teaching secondary school mathematics to Grade Seven students, the researchers noticed that the students have not yet mastered nor do they have automaticity in performing simple addition and multiplication. This prompted them to observe the elementary-level mathematics classes, especially in Grade Three, where multiplication is taught and applied in their lessons. Even some of the students were still using their fingers to do simple addition; they were not yet *mature* in terms of basic mathematics skill.

As an abacus teacher, one of the researchers believes that the abacus is one of the tools that can help hone students' basic mathematics skills such as addition and multiplication. The abacus not only increases the ability of children in performing mathematics calculations, but also develops memory effectively (Gera and Kaur, 2014). The researchers wanted to observe and see how far abacus training can help students to be accurate and speedy in performing addition and multiplication.

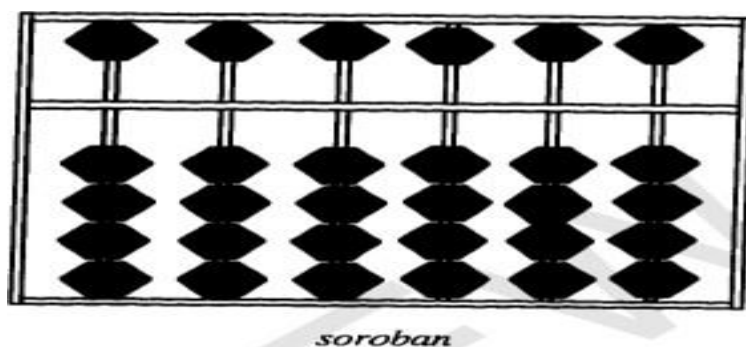


Figure 1. The Soroban (Gilmore, 1997)

In the education setting, Miller and Stigler (1991) espoused the idea that people who have consistently used and mastered the abacus are capable of extremely rapid and accurate mental calculations, with children being able to perform mental calculations by moving the beads in their *mental abacus* (i.e. image of an abacus as imagined by the solver) as they would do on a real one. The abacus is merely a tool; through intensive practice, children are able to imagine and internalize the image of the abacus in their mind, and later on perform mental calculations (“Abacus and its History”, 2007). The statement of problem is “Does the abacus method significantly affect student performance in addition and multiplication?”

The present study is primarily anchored on several theories and ideas that shape its theoretical framework. Piaget’s Theory of Cognitive Development (Woolfolk, 2004) espouses four stages of development wherein the preoperational and concrete operational stages have a significant role in abacus training. The former entails the ability of children to relate objects and symbols, whereas the latter deals with children’s ability to think logically and reversely. In terms of

abacus training, the preoperational stage familiarizes children with the beads of the abacus and how they represent actual amounts and numbers, paving the way for numerals to be coded in their memory as a certain number of beads. Meanwhile, when children progress to the concrete operational stage, development of the mental abacus leading towards mental arithmetic takes place (imagination of abacus).

In addition, Vygotsky's Social Constructivist Theory (Woolfolk, 2004) mentions cultural tools as a means of learning for children. Stigler (1984; 1986) supports this with several mentions of abacus training being heavily supported in East Asian communities; together with the early exposure, consistent practice, and high regard for mathematics learning, children are culturally encouraged to do mathematical computations quickly and accurately at an early age. This idea was supported by several East Asian studies (Wang, et al, 2015; Amamiwa, 2001; Hayashi, 2000). All the aforementioned bind the study, serving as its foundation in the rationale that abacus training, under optimal conditions (i.e. early exposure, long-term consistent practice, etc.), can yield good computational skills: fast and accurate answers.

Method

Thirty 3rd-graders of three elementary schools in Indonesia served as the participants of the study. Two of the schools were in Jakarta and the other school was in Yogyakarta. Of which, 15 were abacus learners who are enrolled in an outside-school abacus course (Level 3) and formed the experimental group. They have mastered the rules in using an abacus. For addition, most of them can do until 2 digits mentally. For multiplication, most of them can do 2-digit times 1-digit mentally.

The experimental group was given additional abacus training for 10 meetings, lasting 45 minutes each session. The Nonequivalent Comparison Group Design (Shadish, Cook and Campbell, 2002) was used. It is a design that consists of giving an experimental and a control group a pretest, followed by a posttest, after the experimental treatment condition (i.e. teaching of the abacus) has been administered to the experimental group. Data gathering was through the pretests and posttests administered to the two groups about addition (until 3 digits) and multiplication (until 2-digits times 2-digits and 3-digits times 3-digits) based on the prescribed Indonesian curriculum. The researchers also utilized observation and interview to gain more insight into the thinking and errors committed by the students. The instruments used in this research consist of the addition and multiplication pretests and posttests, modules (or syllabus) for abacus teaching and learning, and interview guides. For data analysis, Microsoft Excel was used to compute for the means, variances and standard deviations of sets of values. The same software was also used for the following tests of hypothesis: F-test and Student's t-Test. For the analysis of qualitative data from the observations and interviews conducted, content analysis was utilized in an attempt to relate the responses of the students to their test performance and errors committed.

Findings and Discussion

For the quantitative data derived from the pretests and posttests, the goal was to find out if there was a significant difference between the scores and the finish time of the experimental and control group, and between the pretest and posttest data of the experimental group, both at 0.05 level of significance.

Student's t-test was utilized, since the sample size was only 15 (i.e. less than 30).

Comparison Between Control Group and Experimental Group

Addition Scores in Pretest

The F-test reveals that the variances are not equal since $0.0253 < 0.05$; thus, Student's t-test for unequal variances is used. With a p-value of 0.4363, which is more than 0.05, the Student's t-test shows that there is no significant difference between the mean pretest scores of the experimental and control groups in addition. Based on this result, we established that the students' ability in the basic mathematics skill for addition was considered to be similar for both groups. Students in experimental group was using mental abacus for simple addition that involved one to two digits while for two to three digit numbers they was using abacus. For the control group, most of them can use mental arithmetic for the simple addition and the rest using a pen-and-paper method.

In the pretest, the researchers chose to include only a few large numbers. This likely required direct addition without using any rules of abacus for the experimental group; in the case of the control group, there was no need for regrouping. However, it should be noted that in the posttest, most of the items involved larger numbers for both addition and multiplication. The number of incorrect answers in the pretest from the 15 students in the experimental group were 56 items in total, compared to 73 in total for the control group. For simple addition that involved one- and two-digit numbers (i.e. lessons from Grades One and Two), students in the experimental group committed 7 wrong answers, while the control group incurred 13 incorrect responses. For the more complex addition questions that involved more digit span, students in the experimental group made 49 mistakes, while those in the control group had 60 wrong answers. The contents of the pretest were familiar for students of both groups, as these were already taught in the first three grade levels. Hence, this also might have contributed to no significant difference between the pretest scores of both groups. As supported by Piaget (in Woolfolk, 2007, p. 29), children who have existing schemes in their minds can make use of these to make sense of events in their world – in this case, their statistically similar performance in addition, regardless of method used. The pretest results show that both groups have significantly similar ability for addition at the beginning of the study, with addition being familiar to the students since the schemes related to this operation has been formed by grade one or even kindergarten.

Addition Scores in Posttest

The p-value for F-test is 0.1145, which is more than 0.05. Thus, there is insufficient evidence to show that the variances are not equal. It means that the variances are equal. Using t-Test for two samples assuming equal variances, the p-value obtained is 0.2828, which is more than 0.05. Hence, there is insufficient evidence to show that the means are not equal. It means that there is no statistical

difference in the student's performance in posttest scores in addition between the control group and the experimental group after the abacus training. From this result, we concluded that there is no effect in student's performance in terms of accuracy for addition after the abacus training. One factor that might explain this performance for both groups is that the pupils are already in Grade Three, which means that they have acquired the necessary foundational skills related to addition in Grades One and Two. In turn, increasing the digit span should not have posed a significant difficulty in obtaining the sum, regardless of method or device used. Moreover, a research by Wu, et al (2009, p.440) showed that there is no significant difference between abacus and non-abacus users when it came to simple addition questions, and that both groups showed high levels of accuracy. This is parallel to the results obtained for the addition posttest.

For experimental group, even if they know the rules of abacus, they still need more time to practice in addition that involved a longer digit span. Regarding the practice of abacus, Stigler's research in Taiwan (1986) found that "Mental abacus skill was found to develop primarily as a result of practice rather than of selection factors such as socioeconomic status, ability and previous mathematical knowledge" (p. 447). During the training, the researchers did not focus much in doing addition that involved two to three digit numbers, but instead devoted more time in doing addition through *asas* (i.e. repeated addition) since some students in the experimental group had not yet mastered mental abacus for the multiplication of 2-digit by 1-digit numbers, which is a requirement to do abacus quickly for multi-digit multiplication.

Finish Time for Addition in Pretest

As for the finish time in pretest for addition, the F-test shows that the variances are not the same; hence, Student's t-test for unequal variances was used which reveals that the means in terms of finish time have a significant difference for the two groups. Checking the means, the experimental group was significantly faster than the control group in finishing the pretest.

One of the advantage of using abacus is that it removes the need to regroup in every step of addition; students can directly get the result as they start to count from the left to the right, either using abacus or mental arithmetic. This process can help them save time and further master the use of the abacus. This is supported by Miller and Stigler (1991) – to do calculations using the abacus, proper finger technique is a basic requirement to achieve proficiency.

Finish Time for Addition in Posttest

With a probability value of less than 0.05, the F-test reveals that the variances are not equal. Hence, t-Test for two samples assuming unequal variances is used; since the p-value of 0.00033 is less than 0.05, there is sufficient evidence showing that the means are not equal. It means that there is a difference in the finish time for posttest in addition between control group and experimental group. From the results obtained, it can be seen that the experimental group is significantly faster than the control group for the posttest in addition. Some factors that affect their result are as follows: the abacus learners were observed trying to beat the time limit and that they directed their focus in answering the worksheet. Gilmore (1997) notes that such an attitude of students in the

experimental group showed that abacus as a tool can help students prioritize and concentrate, thereby switching their attention to their abacus and worksheet. Using abacus also removes the need for regrouping so they can spend time shorter than others. Shwalb and his colleagues (2004) found in their research how motivation for mathematics is influenced by attending abacus training: whenever the researcher gave the students worksheets, they would directly do the calculations using the abacus. This kind of response was analogized to the students as if seeing their favorite food.

Multiplication Scores in Pretest

The F-test, with a p-value 0.1546 of more than 0.05, shows that the variances are not unequal. Thus, a Student's t-test for assuming equal variances is used to check for the difference between two means. It further reveals that there is no difference between the pretest scores of the two groups in multiplication with the p-value 0.2214 being more than 0.05. Based on this result, the researchers found that the students' ability in the basic mathematics skill for multiplication was considered to be similar for both groups. It might have been due to the lesson on multiplication of two two-digit numbers and three-digit by one-digit numbers being new for all students in both groups. The researchers thus categorized the result as low since the mathematics standard score for Grade Three in their schools is 70 at the minimum. Hence, such multiplication problems posed difficulty for them. Both groups used the conventional multiplication algorithm in solving the multiplication of two two-digit numbers, and multiplication of three-digit and one-digit numbers. Based on this result, the researchers shifted the lessons to focus more on their multiplication, thus giving *asas* practice for addition only.

Multiplication Scores in Posttest

With a p-value of 0.0003 for the F-test, there is sufficient evidence to show that the variances are not equal. Thus, Student's t-test for two samples assuming unequal variances is used, with p-value 0.0003 that is less than 0.05 obtained. Thus, there is sufficient evidence to show that the means are not equal. It means that there is a significant difference in the posttest scores of the two groups, wherein the experimental group obtained higher scores compared to the control group. The short process in multiplication using abacus helps student avoid some errors as compared with the conventional way. In the multiplication of two two-digit numbers using both physical and mental abacus, students only need two steps to find the product, as compared to at least five steps in the conventional way taught in schools. Consider 23×54 . For students who are using the abacus, the first step is $23 \times 5 = 115$ (using mental abacus) and shall be put in the thousands pole. As for the second step, $23 \times 4 = 92$ (using mental abacus) and put it in the tens pole, and at the same time, the students directly apply the partial product in their abacus to get the answer of 1242. In contrast, the conventional way would require students to perform multiplication at least in 5 steps.

For the experimental group, there was a large progress in terms of accuracy using the abacus since students knew about the rules of abacus in addition as one of the important requirements in doing multiplication (Flom and Heffelfinger, 2004). Despite the results showing the progress of students in performing

multiplication, some errors were still observed during posttest and are related to the errors that were found in addition, such as errors about upper bead, omission, and position (Stigler, 1984).

Finish Time for Multiplication in Pretest

As can be seen in the F-test's p-value of 0.3126, which is more than 0.05, the variances are equal. Thus, Student's t-test assuming equal variances is used, revealing a p-value of 0.2106 that is more than 0.05. It is interpreted as having no difference in terms of the means. From this result, it can be said that students in the experimental group and control group have statistically similar finish times for the pretest of multiplication. Based on the observations and the test results, it can be seen that both groups still had difficulty in doing multiplication that involved 2 and 3 digits. Both groups were using conventional way regarding 2 and 3 digits multiplication. Abacus learners in the experimental group have not yet learned about the multiplication of two two-digit numbers and three-digit with one-digit numbers in their abacus class outside of school. Furthermore, for simple multiplication involving one-digit and two-digit numbers (i.e. Grade Two lesson), the students in the experimental group committed a total of 10 mistakes; for multiplication that involved more digits (i.e. Grade Three lesson), the 15 students answered 166 items incorrectly. On the other hand, the control group committed 40 and 199 mistakes for the same categories as mentioned above. From these results, one can see that the number of mistakes increased as more digits were involved in the calculation.

These results were consistent with research conducted by Ashcraft and Koshmider (1991) about the development of children's mental multiplication skill: that the third graders had a 4.3 percent error rate on problems that involved small numbers (e.g. 2×3), but 19 percent error rate on problems that involved larger numbers (e.g. 8×9).

Finish Time for Multiplication in Posttest

For this set of data, the p-value for the F-test is 0.0007 which is interpreted as the data sets having unequal variances. Hence, Student's t-test for two samples assuming unequal variances is used, with p-value of 0.0054 obtained which is less than 0.05. Thus, there is sufficient evidence to show that in terms of finish time, there is a difference for posttest in multiplication between control and experimental groups. From the results, it can be seen that the experimental group was faster than the control group in terms of time spent to finish the posttest. Students who use the abacus do the calculation from left to right, so they can simultaneously get the first partial sum as a part of the product while they are processing the next using the abacus. In the conventional way, students work from right to left, so students can not give an answer until they finish covering the entire process. Abacus learner need only few steps compare with conventional way using by non abacus learner. Abacus learner more focus in doing multiplication for 2 to 3 digits numbers even though some forgot how to do it and created some mistakes.

Comparison of Pretest and Posttest for the Experimental Group Scores in Addition

A p-value of 0.1511, more than 0.05, is obtained from the F-test conducted which translates to unequal variances; hence, there is insufficient evidence to show that the variances are not equal. Student's t-test for two samples assuming equal variances reveals a p-value of 0.4414 which translates to insufficient evidence to show that the means are not equal. It means that there is no significant difference in the students' performance between pretest and posttest in addition. One of the factors that might have affected the result is the limited time for practice especially with the addition of three-digit numbers using the abacus. Although the same rules for addition in abacus apply, the longer digit span might have been a source of unfamiliarity and difficulty to some students. Another possible factor is that the students might have reached their maximum capacity in using the abacus for addition, which implies that although further practice might improve their performance slightly (which was what happened), but such improvement would not be that significantly different anymore compared to non-abacus users. Another factor that might have contributed to such performance, as observed by the researcher, is the presence of errors parallel to those classified by Stigler (1986): errors pertaining to the upper bead, omission or position. The first type of error relates to the upper bead being forgotten to be brought down when performing an operation involving the *small friend*. The second type of error happens when a bead is accidentally moved or knocked by the fingers. The third type of error pertains to confusion over the position of beads, leading to misreading the value (e.g. 7 is read as 2 or vice-versa).

Finish Times in Addition

The F-test, through the p-value of 0.0303 that is less than 0.05, shows that the variances are not equal. Hence, Student's t-test for two samples assuming unequal variances was used wherein a p-value of 0.2119 was obtained, which can translate to insufficient evidence to show that the means are not equal. Based on this result, despite being faster in the posttest for addition, there is no statistically significant difference in the finish time of the experimental group for the pretest and posttest. To shed insight into this result, the researcher contacted an abacus trainer from UCMAS Jakarta to verify the target speed in Level 3. The trainer responded that the target finish time for students in Level 3 abacus is a maximum of 10 minutes for 40 items of addition of two-digit numbers. As for the experimental group, the pretest and posttest administered contained questions of higher competency, requiring the addition of up to three-digit numbers. With the experimental group managing to finish the 40-item pretest and posttest in 6.62 and 5.58 minutes respectively, their speed was faster than the target time set by the abacus training center. Moreover, they were able to complete the tests with advanced competencies in such a short period of time.

Scores in Multiplication

The p-value obtained for the F-test is 0.0174, which means that the variances are not equal. Hence, Student's t-test for two samples assuming unequal variances is used, wherein a p-value of 0.0035, less than 0.05, is obtained – there is sufficient evidence to show that the means are not equal. Thus, there is a

significant difference in the students' performance on accuracy between pretest and posttest for multiplication; moreover, an improvement is noted from pretest to posttest as can be seen from the means. The statistical comparison of pretest (mean score of 65) and posttest (mean score of 82.93) scores in multiplication showed that there is a significant difference between them. As the researchers noted, in the additional abacus training, emphasis was placed on performing fundamental multiplication algorithms to pave the way for multi-digit multiplication. The researchers allocated much time during abacus training on teaching the students the multiplication of two- with one-digit numbers.

During the training, a key problem was that the students were not able to mentally obtain the product of a two- and a one-digit number. Among the 15 students in the experimental group, six of them did not yet use mental abacus for such kind of multiplication. Thus, the researchers took initiative to reteach and train that students on how to do mental abacus here, before they could move to multi-digit multiplication. Thus, much time was spent for helping the students master in mental abacus for two- with one-digit number as a requirement to do multi digit multiplication.

This course of action was in line with the ideas espoused by Stigler (1984). According to him, there is a close relationship between mental abacus users and their capacity in using the abacus physically. Moreover, the more the children practice using abacus, the more they can perform mental abacus. It took four meetings to finish the practice on two-digit by one-digit mental multiplication. Afterwards, the students were asked to use mental abacus for two meetings and followed this up with training on multi-digit multiplication using the abacus for the remaining meetings.

Finish Time in Multiplication

With a p-value of 0.3141 in the F-test, there is insufficient evidence to show that the variances are not equal; hence, it means that the variances are equal. With this, Student's t-test for two samples assuming equal variances is used; it yields a p-value of 0.3746, greater than 0.05, which translates to insufficient evidence to show unequal means. Based on the pre- and posttest results for finish time in multiplication, there was no significant difference between the two tests, even if the average finish time in the posttest (14.8 minutes) was faster than that of in the pretest (17.49 minutes). In consultation with a trainer from UCMAS Jakarta about the target finish time for Level 3 abacus in terms of multiplication, he responded, saying the multiplication examination's standard is 10 minutes for 20 items of multiplying two two-digit numbers. In comparison, students in the experimental group solved 40 items, comprised of two-digit and three-digit numbers, with an average finish time of 14.8 minutes. Paired with the significantly improved mean score in multiplication, the finish time of the students could be seen as satisfactory given the additional questions administered to them, though they could still improve their speed in multiplication through practice.

Furthermore, moving from left to the right in counting using abacus builds a new scheme in the students' mind aside from the usual right to left method as taught in the classroom (i.e. conventional way).

Doing multiplication using the abacus is shorter compared to the conventional way, especially with multi-digit factors. To find the product of two two-digit numbers using abacus, students need only two steps with an assumption that they can do mental abacus already in multiplying two- and one-digit numbers. For example, for the question 53×45 , first, students should choose the correct pole (i.e. the thousands pole) and use mental abacus to place 212 (i.e. the first product, from 53×4). The second step, which is also the last, is to add the next product 265 (i.e. obtained using mental abacus). For the conventional way, students would have to do at least four steps wherein they are multiplying four numbers.

Doing multiplication using the abacus can help students move faster than others who use the conventional way. It is one major factor that makes multiplication for the experimental group faster than the control group. Moreover, more practice on using the abacus and mental abacus for multiplying two- by one-digit numbers is a major contributor to the speed in doing multi-digit multiplication.

In some conversations during the abacus training, the researchers spontaneously asked some students regarding their interest about abacus. Most of them said that they enjoy studying about the abacus because it is not extremely difficult (i.e. once all the rules have been mastered, one can do all the basic mathematics operations, starting from the mastery of addition as the key). Moreover, the students preferred to learn using the abacus (rather than the paper-and-pen method taught in school) because using the abacus is just like playing with beads and the students never felt bored.

Additionally, the students did not feel compelled to join abacus training. This was supported by Hayashi (2000), who stated that abacus training would be useless if children are forced to do it; otherwise, if children want to learn about the abacus and do the practice (i.e. moving the beads, seeing and reading the value of the beads) as fun, they will tend to enjoy learning more and get more profit from their experience.

However, some students said that the amount of homework from school and also from the abacus course sometimes made them feel too tired to finish all the assignments.

Conclusions

The use of abacus to support students in mastering their basic mathematics calculation need more time to practice and master in it (Stigler, 1986). From the context of the current study, the abacus method had more effect in multiplication since the steps in doing multiplication was fewer as compared to the conventional way. It helped students move fast and direct their focus to the problems given to them. Moreover, the use of abacus can encourage students love mathematics since they found it easier to do than the conventional way.

Based on the results of this study, more attention should be given to students who have not yet mastered the basic mathematical operations. Likewise, it is suggested that teachers give emphasis on the preparation of an effective lesson plan in teaching basic multiplication. Since the abacus way for subtraction

is just the opposite of addition, future studies may opt to dwell in this topic. Furthermore, a study about the relation between abacus learners and their performance in problem solving may be an insightful topic for future study.

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