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EXPLORING DIFFICULTIES IN TEXTUAL UNDERSTANDING OF MATHEMATICAL WORD PROBLEMS FROM A PSYCHOLINGUISTIC PERSPECTIVE AND USE OF DRAWINGS

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Abstract
Several studies have reported the importance of textual understanding in the solving of verbal mathematical problems. Therefore, the objective of the present study was to analyze the difficulties the secondary-education students presented in the textual understanding of verbal mathematical problems. An instrument was applied with three verbal problems in three different moments: comprehension of the problem, representation of the situation, and the application of a semi-structured interview. An important factor that could be seen in the responses obtained is that the context played an essential role. It was also obtained that the textual understanding and the cultural context show an important relationship when building a representation of the problem. Finally, according to the results obtained in the study, we suggest that it would be important to encourage students to do previous work to familiarize themselves with the context posed in the problem to be able to reach a solution that satisfies the statement.

Keywords: situation model, textual understanding, verbal mathematical problem

Introduction

Mathematics education has been a fertile field for research and innovation, given that mathematics is fundamental in our society and its applications are present in almost all areas of daily and professional life. However, learning mathematics, especially through the use of mathematical word problems, presents challenges that extend beyond the simple acquisition of numerical concepts and skills.

At the core of mathematics education lies the desire to equip students with the tools necessary to confront and solve problems. As Polya (1945) points out in his work "How to solve it", problem-solving is essentially the act of discovering a path that takes us from an initial situation to a desired final situation. Nevertheless, in the education field, mathematical word problems often add an additional layer of complexity, as the student must translate natural language, with all its ambiguity and richness, into the precise and structured language of mathematics (Cummins et al., 1988).

Psycholinguistics delves into the study of how people understand, produce, and acquire language. When analyzing mathematical word problems from a psycholinguistic perspective, we are faced with the task of unraveling how students process the linguistic information contained in a problem and how they transform that information into mathematical representations. Bruner (1986) reminds us that the act of understanding is not static, but rather involves the active construction of meaning. This construction is strongly influenced by the student's previous experiences, cultural knowledge, and linguistic competence (Ginsburg, 1997).

Over the years, various studies have indicated that errors and difficulties in textual understanding of mathematical verbal problems are not solely due to a deficit in mathematical skills. On the contrary, many of these errors have their origin in difficulties in understanding the text of the problem, in constructing an adequate representation of the situation described, or in connecting that representation with the relevant mathematical procedures (Gerofsky, 1996; Leiss et al., 2010). These difficulties can be exacerbated when problem statements are unclear or formulated ambiguously, leading to erroneous interpretations (Islas et al., 2012).

Notwithstanding, beyond identifying and describing these difficulties, it is crucial to understand their underlying causes. Why do some students face greater challenges than others when tackling mathematical word problems? How do factors such as cultural context, prior educational experiences, or individual language skills influence a student’s ability to understand mathematical word problems?

With this research, we seek to delve into these questions, exploring in depth the interactions between language, thinking, and mathematics. The goal is to shed light on the challenges inherent in understanding mathematical word problems.

Problem statement

Mathematics is a discipline that has been fundamental in the evolution of human thought, both in science and in everyday life. However, one of the most challenging areas within the teaching and learning of mathematics is understanding word problems. Mathematical word problems require not only mathematical skills but also linguistic and cognitive competencies to interpret and translate verbal information into mathematical representations (Polya, 1945; Cummins et al., 1988).

Despite the importance of addressing mathematical verbal problems, many students face difficulties in understanding and solving them (Bruno & Espinel, 2002; Chin & Fu, 2021). These difficulties are not only related to mathematical skills, but also to the ability to build and manage mental models based on the information provided (Johnson-Laird, 1983). The interdisciplinary nature of verbal problems suggests that a psycholinguistic approach may be essential to understanding the roots of these difficulties (Gerofsky, 1996).

The literature has pointed out different factors that can influence the understanding of mathematical word problems. For example, the way statements are presented can affect student understanding (Islas et al., 2012). Likewise, the ability to imagine or visualize situations can improve reading comprehension in mathematical contexts (Glenberg et al., 2004). Furthermore, the construction of situational models, which refer to the mental representation that people make of a text, is crucial for understanding and solving problems (Leiss et al., 2010).

Nevertheless, it is evident that not all students construct these models effectively, which may be a contributing factor to their difficulties.

Bruner (1986) suggests that the way we interpret and understand the world is intrinsically linked to our ability to narrate and construct meaning. This is especially relevant in mathematics, where the ability to construct meaning from verbal statements is essential (D'Amore et al., 1996). Nonetheless, research in mathematics education has not exhaustively explored how linguistic and cognitive skills interact in the construction of meaning in mathematical contexts (Silva, 2014).

Therefore, it is essential to further investigate the difficulties and errors in the textual understanding of mathematical word problems. Understanding these difficulties can offer valuable information to develop effective pedagogical strategies and improve the teaching and learning of mathematics.

Objective

The objective of this research is to analyze the difficulties that secondary education students present in understanding verbal mathematical problems.

Method

This is a qualitative study that allowed us to explore the possible relationships between comprehension skills and the cultural context of students when understanding verbal mathematics problems. In addition, it contributed to providing a stronger understanding of the underlying causes of why students have difficulties in this area and in offering guidelines for improving mathematics education.

Exploratory scope

The exploratory scope of this research involves the initial exploration and understanding of errors and difficulties in the textual understanding of mathematical word problems from a psycholinguistic perspective, without attempting to establish definitive causal relationships or broad generalizations (Hernández et al, 2010). This approach seeks to identify initial patterns and trends in the interaction between language, thinking, and mathematics, as well as generate hypotheses that can be investigated in future studies.

Semi-structured interview

Individual semi-structured interviews provide flexibility, allowing students to express their thoughts and reasoning in their own words while following a predefined question guide to ensure consistency in data collection (Cohen & Manion, 1990).

Participants

The participants in this study were chosen in a stratified manner, specifically considering students who had greater difficulty in understanding the text on the subject matter in their native language (Spanish) and who had poor performance in mathematics. In this way, a sample of six students was selected, three men and three women, who were studying the third grade of secondary education in a school in the City of Puebla, Mexico.

Data collection instrument

Two verbal mathematical problems from existing literature were used, which were modified for this study. Table 1 shows each of the verbal problems.

Table 1. Verbal problems applied to students

Mathematics word problem	
PVM_1	A horse is tied to a lasso that allows it a maximum reach of 2 m, attached to a ring, which moves on a bar in the shape of a right angle whose sides measure 2 m and 4 m. What is the area of the region covered by the horse?
PVM_2	On the beach, Fernando wanted to take a parachute tour, while Luis went diving. When Fernando passed just above Luis, he was 27 m above sea level and Luis was 9 meters deep. How far away are they at that moment?

The instrument was applied in a written format, for the purpose of evaluating textual understanding in three moments:

First moment. The students were asked to read the problem carefully to understand the text.

Second moment. Make a drawing of the situation that is represented in the mathematical problem.

Third Moment. Application of an individual semi-structured interview.

Findings and Discussion

The assessment of the responses to these items allowed us to precisely identify the areas of difficulty in textual understanding and how these difficulties impact each student's ability. All students were asked to draw the situation described in the PVM_1one shown in Table 1. When each of the students' representations were analyzed, the following results were obtained. Two categories were generated, Congruent Drawings (DC) and Non-Congruent Drawings (DNC).

DC Category. They are those graphic representations that have all the elements and data of the problem correctly located. An example of this type of drawing would be the following.

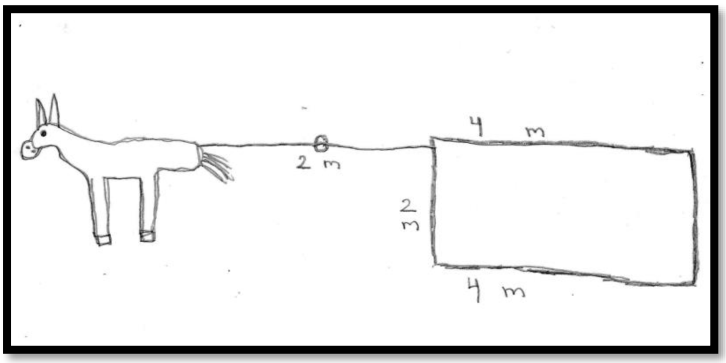

Figure 1. Student S1's drawing of the problem posed in PVM_1

Answers from student S1

I*: Have you witnessed or seen any situation like the one described in the problem?*
S*: Eh... the truth is that once, when I went with my flights and we were trying to take the horses with my grandparents, I left one yellow, I think it is the only time I saw something similar happen.*
I*: What are the characters or objects mentioned in the problem?*
S*: A horse is mentioned, a lasso, what else? what else? a bar that measures 2m by 4m.*
I*: Did any words cause confusion for you when reading?*
S: *Yes, the word shackle, I don't know what a shackle is.*
I*: Regarding your drawing, why do you think it is correct?*
S*: I think it is not because I only tried to do what I understood.*

DNC category. In this category, the representations do not have all the elements described in the word problem and lack the dimensions specified in the problem as well. Five students were placed in this category. You can see drawings like the following in this category.

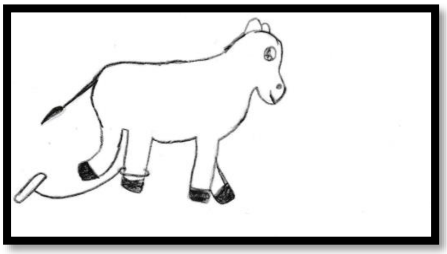

Figure 2. Student S3's drawing of the problem posed in PVM_1

Answers from student S3

I: *Have you witnessed or seen any situation like the one described in the problem?*
S: *No*
I: *What are the characters or objects mentioned in the problem?*
S: *A horse, a 2-meter lasso, and a horseshoe are mentioned.*
I: *Did any words cause confusion for you when reading?*
S: *A shackle.*
I: *Regarding your drawing, why do you think it is correct?*
S: *Well, for the horse, because there is no other way to saddle a horse, and for the horseshoe, well, I believe it is a tool to keep it from leaving.*

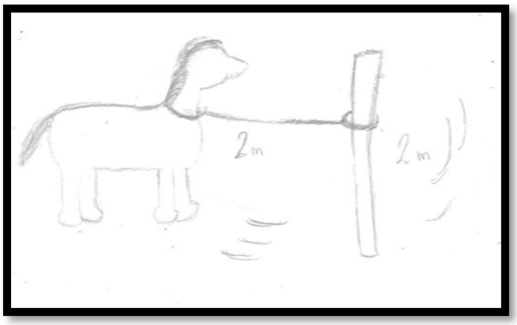

Figure 3. Student S2's drawing of the problem posed in PVM_1

Answers from student S2

I: *Have you witnessed or seen any situation like the one described in the problem?*
S: *No, I only have cats at home, and I don't tie them up.*
I: *What are the characters or objects mentioned in the problem?*
S: *The character is the horse, and the objects are the bar and the shackle.*
I: *Did any words cause confusion for you when reading?*
S: *Yes, the word shackle.*
E: *Regarding your drawing, why do you think it is correct?*
S: *Yes, because I based it on what the problem says.*

In this first problem, it was found that only one student came closest to what was described in the problem, and five students managed to draw the situation described in the problem, but they lacked some elements or the dimensions described in the problem, which in the case of Student S1, who is the one who manages to get closest to the situation of the problem, he could achieve it because he experienced something similar to what he mentions in the problem. It is important to emphasize that cultural context is an important factor so that students can understand the mathematical word problem.

For the $\boldsymbol{PVM_2}$, shown in Table 1, it was analyzed with the same categories of the $\boldsymbol{PVM_2}$and the results obtained were as follows.

DC Category. In this category were placed those drawings that have all the elements and data of the problem correctly located. An example of this type of drawing would be the following.

Figure 4. Student S2's drawing of the problem posed in PVM_2

I: *Have you witnessed or seen any situation like the one described in the problem?*
S: *Yes, on television.*
I: *What are the characters or objects that are mentioned in the problem?*
S: *The characters are Fernando and Luis, and the objects would be the parachute.*
I: *Did any words cause confusion for you when reading?*
S: *No word confused me.*
I: *Regarding your drawing, why do you think it is correct?*
S: *Yes, because it represents what the problem says.*

DNC category. In this category, the representations do not contain all the elements described in the word problem and lack the dimensions specified in the problem. Five students were placed in this category. An example of the above can be seen in Figure 5.

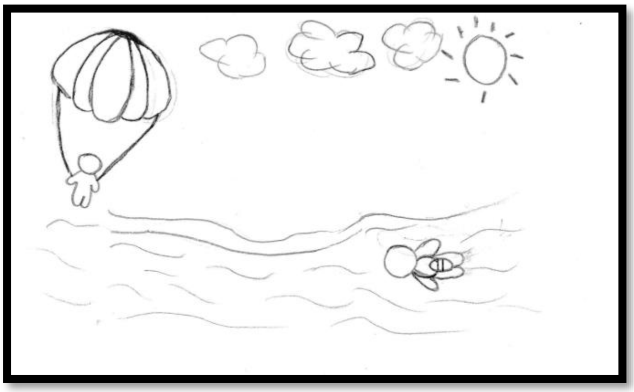

Figure 5. Student S3's drawing of the problem posed in PVM_2

I: *Have you witnessed or seen any situation like the one described in the problem?*
S: *No. But I do know that diving and getting on it are similar.*
I: *What are the characters or objects mentioned in the problem?*
S: *One person parachuting and one person diving*
I: *Did any words cause confusion for you when reading?*
S: *Not that I remember.*
I: *Regarding your drawing, why do you think it is correct?*
S: *Because of the drawn waves, the parachute, and I wanted to give the details of the clouds and the Sun.*

In this second problem, by presenting clearer, unambiguous language with a more familiar context to students, misinterpretations were minimized, and comprehension was improved. In this way, five students managed to successfully understand and draw the situation described in the problem and only one student managed to draw the situations described in the problem, but with some shortcomings such as those shown in Figure 5, where there is no linearity. between the person who is diving and the person who is in the parachute.

Conclusion

The results of this study reflect a strong relationship between textual understanding and cultural context. When students have experienced or have observed a similar situation, they are able to build a mental image with fewer errors and thus improve understanding.

Likewise, when they read and do not understand complete words or sentences, they construct a situation lacking elements present in the text presented to them, which generates confusion and doubts when constructing the mental scheme.

It is evident that difficulties in textual understanding of verbal mathematical problems cannot be attributed solely to students' mathematical abilities. In fact, a lack of familiarity with the context posed in the problem and comprehension skills plays a relevant role in how a student approaches and interprets a problem.

Furthermore, the way students approach word problems is deeply personal and influenced by their previous educational and cultural experiences. This conclusion reinforces the importance of student-centered pedagogy, which seeks to nurture critical thinking and understanding skills beyond simple memorization of formulas or procedures.

Recommendations

Based on the findings and conclusions in this research, the following recommendations are presented to address the difficulties inherent in understanding mathematical word problems.

1. ***Clear Problems.*** When creating or using word problems, it is crucial to ensure that the language used is clear and free of ambiguity. This clarity can minimize misinterpretations and improve accuracy in understanding.
2. ***Promotion of Meta-Cognitive Strategies***. Promote students' ability to reflect on their thinking process. Encourage them to ask questions such as: "What do I understand about the problem?", "Am I interpreting the information correctly?", or "Does my answer make sense in the context of the problem?"
3. ***Culturally Relevant Approach***. Word problems should be contextually relevant to students. Using situations and scenarios that students recognize in their daily lives can improve their connection to and understanding of the problem.
4. ***Collaborative Interaction***. Stimulate group work and discussion among peers. Encouraging students to share and debate their interpretations of a problem can help clarify doubts and diversify understanding strategies.
5. ***Constructive***. When correcting errors, it is essential to not only point out the error but also guide the student through the process of identifying where and how the misinterpretation occurred.

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