

## REALISTIC MATHEMATICS EDUCATION (RME) ASSISTED WITH SCAFFOLDING TO ENHANCE MATHEMATICAL REASONING

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### Abstract

Until recently, the educational curriculum has led to the need for problem-solving through adequate mathematical reasoning. In addition, there is much evidence that problem-solving is achieved through mathematical reasoning. The problem is that primary students' mathematical reasoning is still low. Meanwhile, many empirical studies state that Realistic Mathematics Education (RME) can enhance mathematical reasoning. On the other hand, scaffolding is a solution that helps achieve learning outcomes, including mathematical reasoning. This study examines the impact of RME assisted with scaffolding toward students' mathematical reasoning. The research method was quasi-experimental with a non-equivalent (pre-test and post-test) control-group design. The study sample involved fifth-grade students using a purposive sampling technique. The data analysis used was an independent sample t-test. The study's findings showed that RME assisted with scaffolding, which affected students' mathematical reasoning. Another finding was that students' primary mathematical manipulation as part of mathematical reasoning increased significantly. Therefore, integrating RME and scaffolding can be a new alternative learning strategy to enhance primary students' mathematical reasoning.

**Keywords:** mathematical reasoning, realistic mathematics education, scaffolding

### Introduction

Until recently, the education curriculum, including teaching and learning, is still oriented toward problem-solving (Kaitera & Harmoinen, 2022; Olivares et al., 2021; Siswanto, 2024; Suseelan et al., 2022). Meanwhile, many empirical studies highlight mathematical reasoning as a crucial factor for students' problem-solving success (Hansen, 2022; Hughes et al., 2020; Supriadi et al., 2024). In addition, there is a positive relationship between mathematical reasoning for problem-solving and motivation to learn mathematics (Supriadi et al., 2024). On the other hand, the problem-solving process is driven by two-way interaction by the characteristics of individual students' mathematical reasoning, student roles, and collaborative processes (Hansen, 2022). Mathematical reasoning is a strategy for problem-solving in algebra using a rubric that focuses on the accuracy of logical statements and answers (Hughes et al., 2020). Therefore, mathematical reasoning is a logical



thinking skill that is the basis and an important factor for students' problem-solving success.

Tasks that lead to mathematical reasoning have more direct pedagogical implications than learning by rote (Melhuish et al., 2020; Walkington et al., 2019). Mathematical reasoning provides tremendous potential and opportunities for students to fulfill various aspects of learning, including process standards, concepts, and domains of mathematics (Smit et al., 2023). In other words, mathematical reasoning needs to be assessed thoroughly in the process and product of learning. Dreher (2020) revealed that when students become more proficient in mathematical reasoning, they will be better prepared to explore other studies. Thus, mathematical reasoning supports conceptual understanding and mathematical process standards and prepares students to build the critical thinking needed to explore learning beyond mathematics.

At the primary school level, achieving ideal mathematical reasoning plays an important role in problem-solving skills when using logic and strategies to obtain accurate solutions. Several studies emphasize the importance of integrating mathematical reasoning into the learning process to support primary students' problem-solving skills holistically and effectively (Hughes et al., 2020; Kaitera & Harmoinen, 2022). On the other hand, integrating mathematical reasoning into the primary school curriculum to strengthen students' problem-solving is considered important (Kaitera & Harmoinen, 2022). In addition, mathematical reasoning is needed in problem-solving to bridge the skill of understanding important information by implementing logic and appropriate strategies in solving problems (Hughes et al., 2020).

Based on the results of the Program for International Student Assessment (PISA) study show that the level of mathematical reasoning of students in Indonesia is still in the low category. PISA 2022 results show that the mean score of Indonesian students in mathematics is 366 points, which is lower than the OECD country mean of 472 points (OECD, 2022). More specifically, Romadhon et al. (2024) found that the mathematical reasoning of primary students is inadequate in generalizing statements. If the problem of low mathematical reasoning in primary students is not addressed, Nurlinda et al. (2024) reveal that this will impact students' low problem-solving skills. Therefore, appropriate learning is needed to enhance students' primary mathematical reasoning significantly.

Several researchers have shown Realistic Mathematics Education (RME) as an alternative to enhance mathematical reasoning, including for primary students (Ekowati et al., 2021; Khoirunnisa & Putri, 2022; Nabila & Putri, 2022; Palinussa et al., 2021; Saleh et al., 2018). RME significantly impacts students' mathematical reasoning and communication based on island-based rural contexts (Palinussa et al., 2021). Implementing RME significantly affects students' reasoning on learning in fourth-grade students (Ekowati et al., 2021). Students' mathematical reasoning is enhanced after implementing RME and collaborative learning through video media (Khoirunnisa & Putri, 2022; Nabila & Putri, 2022). Meanwhile, Anggani et al. (2019) explained that implementing RME is not always comprehensively enough to enhance students' mathematical reasoning, especially in primary school education. In addition, according to Melhuish et al. (2020), mathematical reasoning is a higher-order thinking skill. Therefore, additional strategies are needed to

optimize the achievement of mathematical reasoning in addition to implementing RME.

Scaffolding can be an additional strategy in addition to specific learning to help students who experience learning difficulties. Scaffolding is done by providing gradual guidance until students can understand concept understanding independently (Liang & She, 2023). In this case, when the students' level of understanding decreases, the scaffolding strategy is carried out gradually by reducing assistance as the students' understanding deepens (Basir & Wijayanti Dyana, 2020; Calor et al., 2022; Wibowo et al., 2025). More specifically, in problem-oriented learning, Masinading and Gaylo(2022) and Ulya et al. (2023) revealed that scaffolding can help students who experience learning difficulties by providing assistance tailored to student needs. Thus, achieving more optimal student mathematical reasoning when implementing RME assisted with scaffolding is possible.

In recent years, studies on RME and scaffolding have been separately conducted to solve the problem of low mathematical reasoning, especially for primary students. Studies by Ekowati et al. (2021), Palinussa et al. (2021), and Saleh et al. (2018) regarding RME learning could affect enhancing the mathematical reasoning of primary students. Other studies, Basir and Wijayanti Dyana (2020), Ghani et al. (2023), and Jensen et al. (2023), regarding enhancing students' mathematical reasoning can be supported by implementing scaffolding strategies. There is a direct relationship between RME and mathematical reasoning as well as scaffolding and mathematical reasoning, thus opening up the study to explore the potential of these strategies.

Hence, a study that integrates RME assisted with scaffolding to enhance students' mathematical reasoning is still not available and needs to be done by involving primary students. Therefore, it is necessary to formulate the study problem: Does it impact students before and after implementing RME assisted with scaffolding to enhance mathematical reasoning? The study results are expected to provide practical benefits for teachers, namely providing the latest empirical evidence and ways that implementing RME assisted with scaffolding can enhance the mathematical reasoning of primary students with problems. In addition, for policymakers, the study results can be a study that shows that RME assisted with scaffolding, which is an effective breakthrough to enhance mathematical reasoning in primary students.

### ***Mathematical reasoning***

Mathematical reasoning is a skill that underlies many aspects of mathematical learning, especially in the process of problem-solving and making logical decisions. Mathematical reasoning involves cognitive activities to understand, analyze, and draw conclusions based on specific data or patterns (Smit et al., 2023). Meanwhile, mathematical learning in all its content activities requires mathematical reasoning as the basis for students' thinking to draw conclusions or make mathematical statements. Based on the study by Saleh et al. (2018), Mathematical reasoning not only helps students solve mathematics problems but also inspires the development of mathematical knowledge. This indicates that mathematical reasoning is a technical skill and the foundation of mathematical thinking. Mathematical reasoning depends not only on students' innate abilities but also on active learning

approaches, teachers' teaching strategies, and students' involvement in an iterative process of mathematical exploration. Barnes (2019) explained that students' activeness in generating solutions iteratively is the core of an effective learning process, including in the development of mathematical reasoning. This process involves students continuously exploring and enhancing the approach used in solving problems. Ekowati et al. (2021) explained that mathematical reasoning develops through student interaction with challenging mathematics problems, which encourages students to practice using thinking strategies continuously.

### ***Principles of realistic mathematics education (RME)***

Realistic Mathematics Education (RME) emerged as an approach that integrates mathematics education with students' concrete experience, thus helping students to be more focused and able to question problems. Inci et al. (2023) explain that problems in RME can come from the real world, the fantasy world, or the formal world of mathematics as long as students consider the problems realistic. RME is an approach developed by Freudenthal (1968) as a process of deep exploration that involves introducing problem situations, solving problems, generating subjects, reshaping those subjects, and making them meaningful by concretizing them. Akbaş and Yildirim (2024) explained that the connection between mathematics and everyday life is important in helping students associate concrete experiences with mathematical concepts. Therefore, the RME approach can facilitate students' understanding of mathematics by linking mathematical concepts with real-life contexts. Thus, RME makes learning more relevant and meaningful and allows students to build connections between mathematics and students' life experiences, thus supporting a deeper and more applicable understanding. This is in line with the principle of RME, which emphasizes the connection between real things for students. Based on Gravemeijer (1994, 1999), it is explained that there are three basic principles of RME, namely guided reinvention, didactic phenomenology, and self-developed or emergent models (Inci, Peker, & Kucukgencay, 2023). Inappropriate learning approaches and methods can hinder the enhancement of mathematical reasoning. Conventional teaching methods such as lectures, questions and answers, and assignments are still commonly used in mathematical learning in primary schools, so students become passive (Qomario et al., 2020).

### **Method**

The research method used is a quasi-experimental study with a non-equivalent (pre-test and post-test) control-group design, as shown in Figure 1. The research design is carried out by dividing the control group and experimental group, but the design may not involve random assignment to the group either partially or wholly (Creswell & Creswell, 2018).

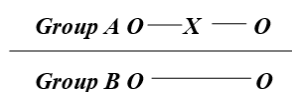


Figure 1. Non-equivalent (pre-test and post-test) control-group design

Based on Figure 1, the study was conducted on two classes, Group A as the experimental class and Group B as the control class. The *X* notation indicates that Realistic Mathematics Education (RME) assisted with scaffolding treatment. Meanwhile, the *C* mark is the class with conventional teaching treatment. Then, the *O* notation is a pre-test given before treatment and a post-test given after treatment to the sample.

The sample involved was fifth-grade, with as many as 48 students in the control and experimental classes. The control class consisted of 23 students each for the control group. Meanwhile, the experimental class consisted of 25 students. A purposive sampling technique was used to select the sample where each individual in the population had an equal chance of being selected as the sample (Creswell & Creswell, 2018).

The instrument used in the study was a mathematical reasoning test. The mathematical reasoning test consists of five essay problems. Scaffolding applied to students is integrated into the RME module teaching material, which is used as a reference for implementing learning. Based on the identification conducted by Roehler and Cantlon (1997) showed that there are five scaffolding that teachers can use: (1) inviting student participation; (2) inviting students to contribute clues; (3) verifying and clarifying student understanding; (4) providing explanations; and (5) modeling the desired behavior (Bikmaz et al., 2010). Before use, the mathematical reasoning test was validated by experts and tested for validity and reliability using SPSS. The validity test results showed values less than 0.05, while the reliability test results showed values of 0.66.

Data collection was carried out using test techniques. Technical tests were carried out by giving mathematical reasoning tests in the form of pre-tests and post-tests. The test was adjusted to the mathematical reasoning indicators, namely: (1) presenting mathematical statements verbally, in writing, pictures, and/or diagrams; (2) making conjectures; (3) mathematical manipulation; (4) proving the truth of a statement; and (5) concluding a statement. Scoring mathematical reasoning uses a 0-4 scale, namely (0) if there is no answer; (1) if the answer shows a fundamental error in principle or concept; (2) if the answer and principle are correct, but the reasoning or calculation is less precise; (3) if the answer is almost perfect with one error; (4) if the answer is correct and complete (Ekowati et al., 2021; Suparman et al., 2021).

Data analysis of students' mathematical reasoning results was carried out in several stages, namely: (1) data collection, (2) test of normality, (3) test of homogeneity, and (4) conduct t-test. Significant differences between the means of the two groups were observed using independent t-tests. An independent t-test was conducted using a statistical analysis computer program called SPSS. It aims to compare the mean of two groups unrelated to each other (independent) to determine whether there is a statistical difference in the variable tested. The study procedure was carried out systematically through several steps, namely: (1) development and preparation of instruments from indicators; (2) study sampling; (3) implementation of pre-test; (4) provision of treatment; (5) implementation of post-test; (6) data analysis; and (7) conclusion/interpretation.

## Findings and Discussion

The results of this study are described in mathematical reasoning, which includes two main things: descriptively and inferentially. Mathematical reasoning descriptively describes the data and mathematical reasoning patterns identified in the study. Meanwhile, mathematical reasoning is inferentially used to generalize the data findings regarding mathematical reasoning and turn them into broader conclusions. Thus, the findings are discussed further.

### *Enhancing mathematical reasoning descriptively*

Using the data from the mathematical reasoning test, we conducted hypothesis testing to find an enhancement in mathematical reasoning between the control and experimental classes. Table 1 shows the summary of mathematical reasoning descriptively.

Table 1. Descriptive statistics

|         | Control Class | Experimental Class |
|---------|---------------|--------------------|
| Mean    | 24.26         | 48.08              |
| SD      | 14.882        | 23.622             |
| Minimum | 0             | 0                  |
| Maximum | 52            | 79                 |

Based on Table 1, the mean score in the control class is 24.26 with a standard deviation (SD) of 14.882, while in the experimental class, the mean score is much higher, at 48.08, with a standard deviation of 23.622. This shows that the experimental class performs better than the control class, although the variability of the values is also greater. The minimum score in both classes was 0. However, the maximum score in the experimental class reached 79, higher than the control class, which only reached 52, reflecting the success of the intervention in the experimental class.

Before hypothesis testing, the data must be tested by normality and homogeneity. A normality test is used to ensure that the data is usually distributed. The homogeneity test is carried out to determine the similarity of variants in the data. The results of the normality test are presented in Table 2, and the results of the homogeneity test are presented in Table 3.

Table 2. Test of normality

| Class              | Statistic | Shapiro-Wilk |       |
|--------------------|-----------|--------------|-------|
|                    |           | df           | Sig.  |
| Control Class      | 0.947     | 23           | 0.258 |
| Experimental Class | 0.931     | 25           | 0.094 |

$\alpha = .05$

In Table 2, the results of the normality test using Shapiro-Wilk show that the data in the control class has a statistical value of 0.947 with a significance of 0.258, meaning that the data is normally distributed ( $p > 0.05$ ). In the experimental class, the Shapiro-Wilk statistical value is 0.931 with a significance of 0.094, indicating that the data is normally distributed ( $p > 0.05$ ). Thus, both groups met the normality assumption for further statistical analysis.

The results of the homogeneity test in Table 3 show that the variance between groups in this study is significantly different. Based on the mean, the Lavene Statistic value is 6.084 with degrees of freedom ( $df_1=1$ ,  $df_2=46$ ) and a significance of 0.017, which indicates the difference in variance is significant at the 5% confidence level. Meanwhile, based on the median, the Lavene Statistic value is 4.121 with a significance of 0.048, indicating a significant difference in variance between groups. This indicates that the data is not homogeneous, so more attention is needed to choose the following analysis method.

Table 3. Test of homogeneity

|                 | Lavene Statistic | df1 | df2 | Sig.  |
|-----------------|------------------|-----|-----|-------|
| Based on Mean   | 6.084            | 1   | 46  | 0.017 |
| Based on Median | 4.121            | 1   | 46  | 0.048 |

$\alpha = .05$

### ***Enhancing mathematical reasoning inferentially***

After the normality and homogeneity test is fulfilled, the next thing to do is an independent sample t-test on the control and experimental classes. The test results of the control class are presented in Table 4.

Table 4. Result independent sample t-test control class

|                        | Control Class | n  | Mean  | Std. Deviation | Sig.  |
|------------------------|---------------|----|-------|----------------|-------|
| Mathematical Reasoning | Pre-test      | 23 | 11.61 | 7.715          | 0.002 |
|                        | Post-test     | 23 | 23.61 | 15.117         | 0.002 |

$\alpha = .05$

The results of the data analysis showed that in the control class, there was an increase in the mean value of students' mathematical reasoning from the pre-test to the post-test. The mean pre-test score was 11.61 with a standard deviation of 7.715; in the post-test, it increased to 23.61 with a standard deviation of 15.117. The significance test yielded a p-value of 0.002, indicating that the increase was statistically significant. This indicates that although the control class was not given special treatment, there was a significant development in students' mathematical reasoning. The experimental class test results are presented in Table 5.

Table 5. Result in independent sample t-test experimental class

|                        | Experimental Class | n  | Mean  | Std. Deviation | Sig.  |
|------------------------|--------------------|----|-------|----------------|-------|
| Mathematical Reasoning | Pre-test           | 25 | 16.48 | 8.357          | 0.000 |
|                        | Post-test          | 25 | 48.08 | 23.622         | 0.000 |

$\alpha = .05$

The results of data analysis in the experimental class showed a significant increase in students' mathematical reasoning. The pre-test mean value of 16.48 with a standard deviation of 8.357 increased to 48.08 in the post-test with a standard deviation of 23.622. The statistical significance of 0.000 in both tests indicated that the difference between the pre-test and post-test was statistically significant,

showing the effectiveness of the learning intervention provided. This shows that the RME approach assisted with scaffolding effectively enhances students' mathematical reasoning.

The results of this study provide new empirical evidence that RME assisted with scaffolding in enhancing students' mathematical reasoning experimentally. This study is consistent with previous studies on implementing learning related to students' mathematical reasoning in an experimental study design (Ekowati et al., 2021; Palinussa et al., 2021). The researchers explained that RME is a learning approach that affects students' mathematical reasoning (Ekowati et al., 2021; Khoirunnisa & Putri, 2022; Nabila & Putri, 2022; Palinussa et al., 2021; Saleh et al., 2018). In addition, scaffolding optimally affects students' mathematical reasoning achievement by applying scaffolding when students face difficulties in learning gradually. On the other hand, scaffolding is given by adjusting student needs (Masinading & Gaylo, 2022; Ulya et al., 2023).

Another finding in this study is a significant increase in mathematical manipulation skills compared to other mathematical reasoning components. However, this shows that mathematical reasoning components other than mathematical manipulation still need to be enhanced to help students overcome difficulties in solving complex problems (Gultom et al., 2022; Smit et al., 2023). In particular, the mathematical reasoning component that focuses on proving the truth of a statement is still relatively low. The proof is divided into two parts, formal and informal, using examples, concrete objects, and examples and concrete objects. Formal proof is possible for students with higher education levels that require generalizing the statements and abstraction skills (Romadhon et al., 2024). So, it will provide an important early experience for primary students to build meaningful mathematical ideas and prepare them for formal proofs at higher education levels (Amir & Amir, 2021).

Students also had difficulty proving the truth of a mathematical statement through logical arguments and strong evidence. At the same time, this component is essential because it involves problem-solving and deep thinking processes. According to Liang and She (2023), the low level of this component indicates that students may face difficulties in understanding, solving, and applying basic mathematical concepts critically. Mathematical reasoning is important in problem-solving because it can help students understand and apply more complex concepts in real situations. Therefore, according to Kaitera and Harmoinen (2022), integrating mathematical reasoning and problem-solving in the curriculum is considered important to strengthen problem-solving; that is, when students are faced with problems, mathematical reasoning is processed when students solve problems.

Smit et al. (2023) stated that if you want to enhance mathematical reasoning significantly, enter the learning process. Therefore, it is recommended that RME be designed to assist with scaffolding learning that focuses on problem-solving. This approach shows great potential in significantly enhancing students' mathematical reasoning. Appropriate scaffolding is very influential in ensuring students get the appropriate help they need to solve mathematical problems (Masinading & Gaylo, 2022; Ulya et al., 2023). Effective scaffolding helps students overcome difficulties in understanding complex mathematical concepts and guides them in a critical and logical thinking process. Thus, integrating RME and scaffolding is expected to



optimize learning outcomes, especially in developing students' problem-solving to get better product mathematical reasoning results.

Based on this study, theoretical, methodological, and practical contributions can be used by processing and integrating the study results to produce meaningful and applicable recommendations. In terms of theoretical contribution, the results of this study support previous findings which show that implementing the RME approach can significantly enhance mathematical reasoning (Anggani et al., 2019; Ekowati et al., 2021; Khoirunnisa & Putri, 2022; Nabila & Putri, 2022; Palinussa et al., 2021; Saleh et al., 2018). In addition, early findings show that assisted scaffolding can be integrated with RME to enhance students' mathematical reasoning.

This study also provides a methodological contribution, mainly because it was conducted with a quantitative experimental study design. This is different from most previous studies that tend to use qualitative, descriptive quantitative, or action research designs (Anggani et al., 2019; Akbaş & Alan, 2022; Khoirunnisa & Putri, 2022; Nabila & Putri, 2022; Revina & Leung, 2021). This study evaluates students' mathematical reasoning quantitatively and describes mathematical reasoning in depth by analyzing the learning activities applied.

In addition, related to practical contributions in education, the results of this study are helpful as a reference for teachers in designing teaching materials, especially in compiling learning steps by implementing RME assisted with scaffolding, which is proven to enhance students' mathematical reasoning. In this case, RME can be considered and chosen as a learning approach that can encourage student achievement in the academic field (Aksu, 2021; Gübbük & Uygün, 2024; Qomario et al., 2020; Tong et al., 2022) and use scaffolding to help students who experience difficulty in the learning process (Basir & Wijayanti, 2020; Calor et al., 2022; Liang & She, 2023; Ulya et al., 2023; Wibowo et al., 2025). Therefore, policymakers can use the study findings to support the implementation of RME assisted with scaffolding as a practical learning approach. This approach can be implemented to enhance students' mathematical reasoning equally.

## Conclusion

The study results show that realistic mathematics education assisted with scaffolding affects students' mathematical reasoning. This finding supports previous studies on the impact of realistic mathematics education on mathematical reasoning and adds evidence to the importance of scaffolding in facilitating students' difficulties. Nevertheless, this study has some limitations, namely the low proving the truth of a statement of students. Thus, further research is recommended to examine the implementation of realistic mathematics education assisted with scaffolding that focuses on problem-solving. Therefore, a learning approach that combines realistic mathematics education and scaffolding can be used as a reference in curriculum development and mathematics teaching strategies, especially in primary schools, to enhance students' mathematical reasoning more optimally.

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