International Journal of Applied Sciences and Smart Technologies

Volume 2, Issue 1, pages 45–58

p-ISSN 2655-8564, e-ISSN 2685-9432

Area Under the Curves, Volume of Rotated, and Surface Area Rotated About Slanted Line

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(Received 29-06-2019; Revised 05-08-2019; Accepted 05-08-2019)

Abstract

In Calculus, a definite integral can be used to calculate the area between curves and coordinate axes at certain intervals; surface area and volume formed if an area is rotated against the coordinate axis. Problems arise if you want to calculate the area of the area bounded by a curve and a line that does not form an angle of 00 or 900 to the coordinate axis, as well as the calculation of the volume of objects and the surface area of a revolution axis of rotation is a slanted line. By using existing definitions, a formula is developed for this purpose. This paper produces a finished formula, to make it easier for calculus users who do not want to know the origin method. The method used is only the method commonly used. What's new is that this formula hasn't been published in tertiary institutions or universities in Indonesia.

Keywords: the area under the curve, the volume of the rotating object, the surface area of the rotating object, the slanted line axis

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Volume 2, Issue 1, pages 45–58
p-ISSN 2655-8564, e-ISSN 2685-9432

1 Introduction

Mathematics is one of the subjects given to students from elementary school to university. One part of Mathematics is Calculus. Calculus has been given since high school to university, especially the Study Program related to Mathematics and Science. Discussion on Calculus consists of functions, limits, derivatives, derivative applications, anti-derivatives, indeterminate integrals, application of definite integrals, and integration techniques [3].

Integral is one of the sciences in the field of mathematical analysis that continues to grow rapidly, both theoretically and its applications. The application of the integral includes calculating the area of the average, the volume of the rotating object, and the area of the rotating surface. The volume of the rotating object and the swivel surface area is obtained by rotating the area of the flat plane rotated on a rotary axis [6].

Using the theory of calculating the area bounded by curves, x axis or y axis in interval $[a, b]$, commonly studied in Calculus, likewise the volume of a rotating object occurs if a curve is rotated 360^0 against the x axis or y axis, or the surface area of an object that occurs when an area is rotated against the x axis or y axis [1,3]. Problems arise, if we want to calculate the area under the curve and a sloping straight line, that is the line that forms an angle β to the positive x axis with $0^{\circ} < \beta < 90^0$, the area of the rotating object and the volume of the rotary object if an area is rotated against a slanted line as shown in Figure 1 [4,6].

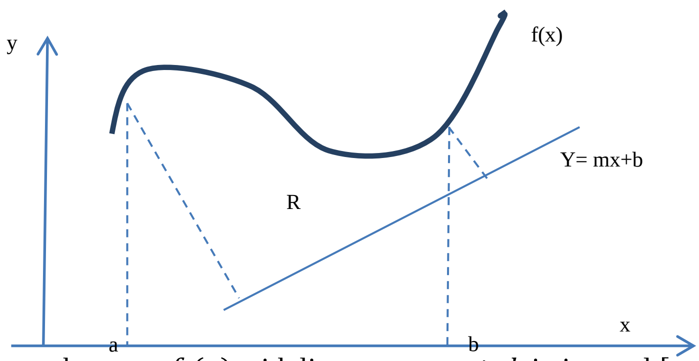

Figure 1. Area-area between $f\ (x)$ with line $y\ =\ mx\ +\ b$ in interval $[a, b]$

This paper is intended to formulate a formula to calculate the area under a curve, the volume of a rotating object, and the area of a rotating object against a slanted line. The position of this paper is complementary to the existing one and also to make it easier for calculus users to calculate the area under the curve up to the slanted line, the surface area of the rotating object against the slanted line, and the volume of the rotating object towards the slanted line.

2 Research Methodology

Following are a few things to keep in mind in coding equations for submission to This article is a literature review. The steps of writing are as follows

1. Reviewing Calculus textbooks, Purcell, Thomas, and James Stewart, an inseparable part of the application to calculate the area under the curve until one of the coordinate curves, rotating objects that occur when rotated in the area passed by the coordinates.
2. From Purcell, Thomas, and James Stewart's books, only in James Stewart's book found an assignment to determine the area under the curve towards the slanted line.
3. From this, developed to determine the surface area and volume that occurs if the area is rotated 3600 towards the slanted line.
4. Subsequently tested on students participating in the Calculus class majoring in Industrial Engineering and Information Engineering.

Due to time constraints, this article only contains additions to calculate the area under the curve of a slanted line, surface area and volume of objects that occur, if there is an area rotated 3600 towards a slanted line.

3 Results and Discussions

By using Integral Calculus we can calculate the area as in Figure 2. Note the area R as shown by Figure 2. The region R is bound by the x axis, the lines $x = a$ and $x = b$, and the curve having the equation $y = f(x)$, with f a continuous function at a closed interval $[a, b]$. To easily fetch $x\ (x) > 0$, for all x in $[a, b]$. We will assign a value of A as the size of R.

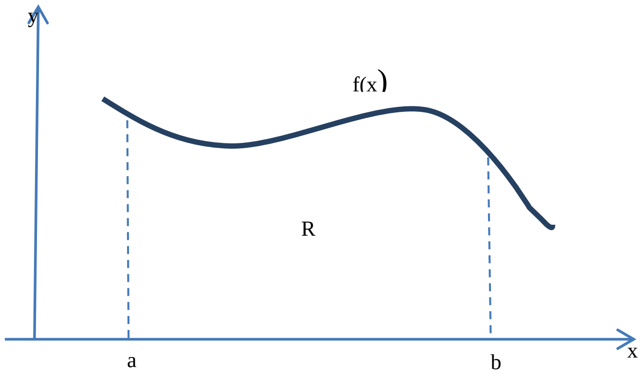

Figure 2. Area under the curve until the x axis

Firstly specify the polygon area in R. The closed hose $[a, b]$ is divided into n part of the interval of the part. To make it easier, each section hose has the same length, for example Δx. So $\Delta x = (b - a)/n$. State the endpoint of each of these intervals by $x_1, x_2, x_3, x_4, \dots, x_n$; with $x_0 = a$, $x_1 = a + \Delta x$; $x_2 = a + 2.\ \ \Delta x, \dots, x_i = a + i.\Delta x, \dots, x_n = a + (n).\Delta x = b$. Name the first part of the hose expressed by $[x_{i-1}, x_i]$ since continuous f function at closed hose $[a, b]$ then f continuously at each part interval [2].

According to the extreme value theorem, there is a number in each interval, at that point f reaches the absolute minimum. Suppose that in the interval the $i - th$ part of this number is c_1 such that $f(c_1)$ is the absolute minimum value of f in the interval section $[x_{i-1}, x_i]$. Note n rectangular pieces, each of which has a width of Δx unit length and height $f(c_1)$ unit length. Suppose Sn area unit states the total area of n rectangular pieces, then:

$$S_n = f(c_1).\Delta x + f(c_2).\Delta x + \cdots + f(c_i).\Delta x + \cdots + f(c_n).\Delta x$$

or

$$S_n = \sum_{i=1}^{n} f(c_i)\,.\,\Delta x$$

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Addition to the right hand side of the formula above gives the area of n rectangles.

Definition 1 :

Suppose the function f is continuous in the closed $interval$ $[a, b]$, with $f(x) > 0$ for each x in $[a, b]$, and that R is the area bounded by the curve $y = f(x)$, x axis, and lines $x = a$ and $x = b$. The hose $[a, b]$ is divided into n interval pieces, each in length $= (b - a)/n$; and the interval of the second part is expressed by $[x_{i-1}, x_i]$ $f(c_1)$ is the absolute minimum value of the function in the interval part i, the size of area R is given by [5]

$$A = \lim_{n \to \sim} \sum_{i=1}^{n} f(c_i) \,.\, \Delta x$$

Definition 2 :

If f is a function defined in the closed interval $[a, b]$ then certain integrals f from $a - b$, $\int_a^b f(x)dx$ are give by $\int_a^b f(x)dx = \lim_{\Delta x \to 0} f(x_i)$, if the limit exists.

Definition 3 :

Suppose the function f is continuous in the closed interval $[a, b]$, with $f(x) > 0$ for each x in $[a, b]$, and that R is the area bounded by the curve $y = f(x)$, x axis, and lines $x = a$ and $x = b$, then size A of area R is given by:

$$A = \int_a^b f(x)dx = \lim_{\Delta x \to 0} f(x_i)\Delta x$$

Suppose the function f is continuous at the closed interval $[a, b]$ with $f > 0$ for each x in $[a, b]$, and that R is a solid object obtained by rotating the area bounded by the curve $y = f(x)$, the x axis, and the lines $x = a$ and $x = b$ with respect to the x axis, then the size V is the volume of area R given by

$$V = \pi \int_a^b f(x)^2 dx = \mathop{lim}_{\Delta x \to 0} \sum_{i=a}^{n} nf\,(x_i)^2 \Delta x$$

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Definition 4 :

Suppose the function f is continuous on the closed interval $[a, b]$, with $f(x) > 0$ for each x in $[a, b]$, and that R is the surface obtained by rotating 3600 regions which are limited by the curve $y = f(x)$, x axis, and lines $x = a$ and $x = b$ with respect to x axis, then size A from area R is given by [7] :

$$A = 2\pi \int_a^b f(x)ds = \lim_{\Delta x \to 0} \sum_{i=a}^{n} 2\pi f(x_i)\Delta s$$

From the above definitions, starting discussion with including Area under the curve until the slanted line, surface area of rotating objects 360^0 against the slanted line , and volume of rotating objects 360^0 against the slanted line.

a. Area under the curve until the slanted line $y = mx + b$, as shown in Figure 3:

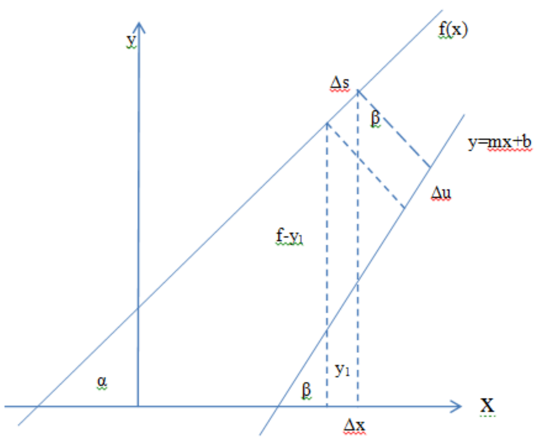

Figure 3. Small pieces perpendicular to the line $y = mx + b$

Take the area around , Δs, enlarged like Figure 4

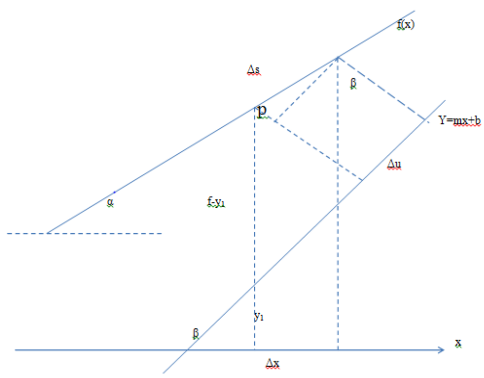

Figure 4. Section of the rectangle area Δu

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Angle $p = 90^0 + \alpha - \beta so$ $\sin\ (90^0 + \alpha - \beta) = \frac{\Delta u}{\Delta s}$, if $\sin\ (90^0 + \alpha - \beta) = \cos\ (\alpha - \beta)$, then

$$\Delta s = \frac{\Delta u}{\cos(\alpha - \beta)} \tag{1}$$

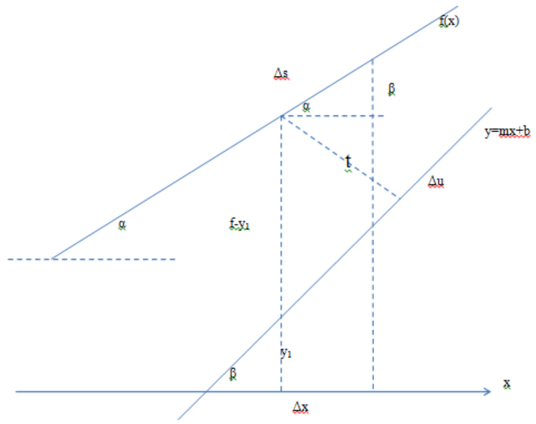

Figure 5. The area around Δu is enlarged

From Figure 5,

$$\cos\alpha = \frac{\Delta x}{\Delta s} \tag{2}$$

Subtituted (1) to (2), we get

$$\Delta u = \Delta x.\frac{\cos(\alpha - \beta)}{\cos\alpha} \tag{3}$$

Because $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$ so (3) become

$$\Delta u = \Delta x.\frac{cos\ \alpha.cos\ \beta + sin\ \alpha.sin\ \beta}{cos\ \alpha}$$

$$\Delta u = \Delta x.\{\cos\beta + \tan\alpha.\sin\beta\}$$

$$\Delta u = \Delta x.\cos\beta.\{1 + \tan\alpha.\tan\beta\} \tag{4}$$

Hence $\tan\alpha$ is a gradient of tangent, then $\tan\alpha = f'$ and $\tan\beta = m$ so (4) become

$$\Delta u = \Delta x.\cos\beta[1 + f'.m] \tag{5}$$

From Figure 5, $\cos\beta = \frac{1}{f-y_1}$ or

$$t = (f - y_1).\cos\beta \tag{6}$$

If $\Delta A = t.\Delta u$, then subtitued (5) and (6) get

$$\Delta A = (f - y_1).\cos\beta\,.\Delta x\,[1 + f'.m]$$
$$\Delta A = (f - y_1).\cos^2\beta\,[1 + f'.m].\Delta x.$$

Because $\cos^2\beta = \frac{1}{1+\tan^2\beta}$ and $\tan\beta = m$, so

$$\Delta A = (f - y_1).\frac{1}{1+m^2}.[1 + f'.m].\Delta x.$$

From definition are we get

$$A = \int_a^b \frac{1}{1+m^2}.(f - y_1).[1 + f'.m]dx \tag{7}$$

This formula use for calculate area the curve until the slanted line.

b. Area of Rotating Objects Against The Slanted Line

If the slice Δs is rotated 360^0 against the line $y = mx + b$, a thin cylinder is formed with t as the radius and Δs as the height, so that the surface area is

$$\Delta A = 2\pi.t.\Delta s$$

With subtitute (2) , (6), $\cos\beta = \sqrt{\frac{1}{1+m^2}}$ and $\cos\alpha = \frac{1}{\sqrt{1+(f')^2}}$ get

$$A = \int_a^b \frac{2\pi}{\sqrt{1+m^2}}(f - y_1).\sqrt{1 + (f')^2}\,dx \tag{8}$$

This formula is used for calculate area of rotating 360^0 against the slanted line.

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c. Volume of Rotating Objects

Now discuss for to get formula of volume of rotating objects 360^0 against the slanted line. If the slice Δs is rotated 360^0 against the line $y = mx + b$, it will form a thin cylinder with t as the radius and Δu as the height, so the volume is

$$\Delta V = \pi . t^2 . \Delta u$$

With subtitued (4), (6) , and $cos\, \beta = \sqrt{\frac{1}{1+m^2}}$ get

$$\Delta V = \pi . (f - y_1)^2 . cos^3 \beta . (1 + f' . m) . \Delta x$$

and

$$V = \int_a^b \frac{\pi}{(1+m^2)^{\frac{3}{2}}} (f - y_1)^2 . (1 + f^\wedge{}' . m) dx \tag{9}$$

This formula is used for calculate volume of rotating 360^0 against the slanted line.

d. Example

Now for example, the area will be determined by the line AB, as shown in Figure 6. The line $y = 2x + 3$ and the line perpendicular to $y = 2x + 3$ drawn from point (1,8) and B (3,10).

Solution:

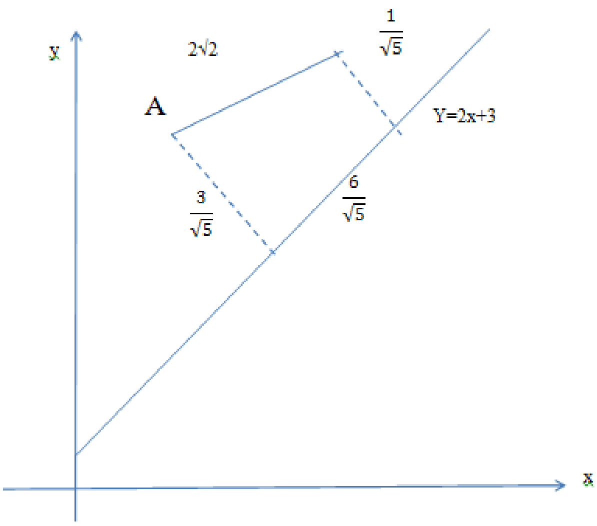

Figure 6. Visualizes the sample count

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Equation of line of AB is $y = x + 7$, gradient $m = 1,\ Y = 2x + 3$ so $f' = 2$

Area counted using formula (7)

$$Area = \int_1^3 \frac{1}{1+2^2}.(x + 7 - 2x - 3).(1 + 2.1)dx$$

$$Area = \int_1^3 \frac{1}{5}.(4 - x).3dx$$

$$Area = \frac{3}{5}.\int_1^3 (4 - x)dx$$

$$Area = \frac{3}{5}.(4x - \frac{1}{2}x^2)_1^3$$

$$Area = \frac{3}{5}.[\left(4.3 - \frac{1}{2}.3^2\right) - \left(4.1 - \frac{1}{2}.1^2\right)]$$

$$Area = \frac{3}{5}.\left[12 - \frac{9}{2} - 4 + \frac{1}{2}\right] = 2{,}4$$

The form of wake occurs is trapeze with measure of edge parallel is $\frac{1}{\sqrt{5}}$ and $\frac{3}{\sqrt{5}}$ and height $\frac{6}{\sqrt{5}}$.

Area of trapeze is ½ sum of edge paralels accrossed by heightso given

$\frac{1}{2}.\left(\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{5}}\right).(\frac{6}{\sqrt{5}}) = \frac{1}{2}.\frac{4}{\sqrt{5}}.\frac{6}{\sqrt{5}} = \frac{24}{10} = 2{,}4$

For the area rotated against the line $y = 2x + 3$, the object that occurs is the surface of the cone. The area can be calculated using the formula of a large cone blanket area minus the size of a small cone blanket. The cone blanket area is calculated by formula $A = \pi.r.s$

Where A is the area of a cone blanket r is the radius of the cone base s is a cone painter line (hypotenuse).

For the example of this count the large cone base radius is $\frac{3}{\sqrt{5}}$ and s is $4\sqrt{2}$ and the small cone base radius is $\frac{1}{\sqrt{5}}$ and the painter's line length is $\sqrt{2}$.

$$A = \pi.\frac{3}{\sqrt{5}}.4\sqrt{2} - \pi.\frac{1}{\sqrt{5}}.\sqrt{2} = \frac{8}{5}\pi\sqrt{10}$$

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Using formula 8 we get

$$A = \int_1^3 \frac{2\pi}{\sqrt{1+m^2}}.(f - y_1)\sqrt{1+(f')^2}\,dx$$

$$A = \int_1^3 \frac{2\pi}{\sqrt{1+2^2}}(x + 7 - 2x - 3).\sqrt{1+(1)^2}dx$$

$$A = \frac{2\pi\sqrt{2}}{\sqrt{5}}\int_1^3 (4 - x)dx$$

$$A = \frac{2\pi\sqrt{10}}{5}.-1(4-x)^2]_1^3$$

$$A = -\frac{2\pi\sqrt{10}}{5}[(4-3)^2 - (4-1)^2] = \frac{8\pi\sqrt{10}}{5}$$

While he volume of objects that occur is a cone that is stuck. Using formula $V = \frac{1}{3}\pi.r^2.t$, we count volume big cone minus volume of a little cone.

$$V = \frac{1}{3}\pi.(\frac{3}{\sqrt{5}})^2.\left(\frac{9}{\sqrt{5}}\right) - \frac{1}{3}\pi.(\frac{1}{\sqrt{5}})^2.(\frac{3}{\sqrt{5}})$$

$$V = \pi.\frac{27}{5\sqrt{5}} - \pi.\frac{1}{5\sqrt{5}}$$

$$V = \pi.\frac{26}{5\sqrt{5}}$$

Using formula 9 get

$$V = \int_1^3 \frac{\pi}{(1+2^2)^{\frac{3}{2}}}.(4-x)^2.(1+2.1)dx$$

$$V = \int_1^3 \frac{3\pi}{5\sqrt{5}}.(4-x)^2dx$$

$$V = \frac{3\pi}{5\sqrt{5}} . -\frac{1}{3}(4-x)^3]_1^3$$

$$V = -\frac{\pi}{5\sqrt{5}} . [1^3 - 3^3]$$

$$V = \frac{26\pi}{5\sqrt{5}}$$

4 Conclusions

From the discussion of examples of counts, it can be seen that using formulas 7, 8, and 9 are not different by using the formula of flat building and space that is commonly used. Thus the area under the curve to the straight line is known not horizontally or vertically and the rotating surface area and the volume of rotary matter can be calculated using formulas (7), (8), and (9). Suggestions for future researchers, can conduct learning research, valid or not the formula.

Acknowledgements

The author would like to thank his colleagues who have helped prepare everything needed so that this paper can be realized.

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International Journal of Applied Sciences and Smart Technologies
Volume 2, Issue 1, pages 45–58
p-ISSN 2655-8564, e-ISSN 2685-9432

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