# APPLICATION OF KLEIN-4 GROUP ON DOMINO CARD

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**Abstract**
Klein-4 or Klein-V group is a group with four elements including identity. The binary operation on Klein-4 group will produce identitiy if operated to it self and produce another non-identity element if operated to another non-identity element. The focus of this paper is to explain the principle of Klein-4 group on Domino card and also find all complete possible elements. Domino card is a set of 28 cards which the surface of each card divided in two identic boxes contain combination of dots as pattern. The final part of this research is to find out all possible combination of cards which can be used as elements of a Klein-4 groups.

**Keywords**: Klein, Domino

## 1. Introduction

Group is a set of nonempty elements with a well define binary operation which satisfy conditions for binary operation are associative, has identity and inverse element, and close under binary operation (see : [1]). We notice a group G with binary operation $*$ as (G,$*$).

**Definition 1.1.** Let (G,$*$) be a group. Then G is an **abelian** group if elements of G are commutative under $*$.

For our convenience, let us denote $\{p\}^k$ be $k-th$ repetitions of $p$ under the operation of binary operation $*$. For example

$$\{p\}^2 = p * p, \quad and \quad \{p\}^4 = p * p * p * p.$$

Let $p$ and $q$ are two numbers such that $0 \leq p \leq 6$ and $0 \leq q \leq 6$. We construct an abreviation with eqivalent meaning as $0 \leq p, q \leq 6$ to help us in simplification writing.

**Definition 1.2.** Let (G,$*$) be a group, and $p, q, e \in G$, with $e$ is the identity. We say $p$ is **generator** of $q$ if there exists a positive integer $n$ such that $\{p\}^n = q$. In this case it is not surprise to make an understending that $p$ generate $q$. Furthermore, we say a positive integer $m$ be the **order** of $p$ if $\{p\}^m = e$.

**Definition 1.3.** Let (G,$*$) be a group. Then we say $G$ is a **cyclic group** if there exists an element $p \in G$ and $p$ generate all elements of G.

A Klein-4 group or Klein-V group is a group with four elements including identity. Klein-4 with a binary operation $\bullet$ and $e$ as identity, will produce $e$ if an element operated to it self and produce it self if operated with $e$. If two different non-identity elements are operated by $\bullet$, will produce the another non-identity element. For example, let $(G, \bullet) = \{e, a, b, c\}$ as a Klein-4 group. Then, the result of the operation over $\bullet$ represented as:

| $\bullet$ | $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e$ | $e$ | $a$ | $b$ | $c$ |
| $a$ | $a$ | $e$ | $c$ | $b$ |
| $b$ | $b$ | $c$ | $e$ | $a$ |
| $c$ | $c$ | $b$ | $a$ | $e$ |

Table 1: Klein-4 group

Since for any $p, q \in$ Klein-4 group with binary operation $\bullet$ implies $p \bullet q = q \bullet p$, then the Klein-4 group is an abelian group.

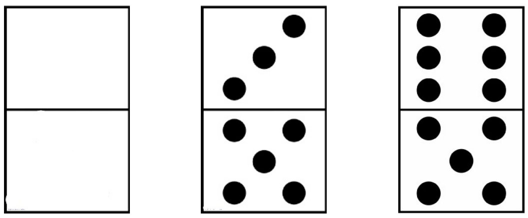
**Theorem 1.1.** *Let (G,$\bullet$) be a Klein-4 group with $e$ as identity. Then the cyclic subgroup of (G,$\bullet$) is only contain identity $e$ and another element such that maximum order of cyclic subgroup of (G,$\bullet$) is 2.*

*Proof.* Let $p \in$ G. If $p = e$, then {p} is a trivial cyclic subgroup of G. If $p \neq e$, then $p \bullet p = e$. Hence, for any $n, m \in \mathbb{Z}$ we have $p \bullet p = (p^2)^n = e^n = e$. Let $m$ be and odd number. Then we always have $n$ such that $m = 2n + 1$. This fact allowed us to have $p = p^m$, since $p^m = p^{2n+1} = p^{2n} * p = e * p = p$. Therefore, $p$ is the generator of $\{p\}$ and $\{e, p\}$. Then we have prove our assertion. $\square$

Domino cards is a set of cards with every card's surface devided in to two boxes and contain the combination of 0 up to 6 dots. A set of domino card contains 28 cards. We can represent the combination of dots on the card's surfaces as paired of positive integer number as:

$(0,0), (0,1), (0,2), \cdots, (0,6), (1,1), (1,2), \cdots, (1,6), (2,2), (2,3), \cdots, (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6).$

We also need to remind that for domino card, $(p, q) = (q, p)$ where 0$\leq$ $p, q$ $\leq$6. For example,



(0, 0)  (3, 5)  (5, 6).

## 2. KLEIN-4 GROUP PRINCIPLE ON DOMINO CARD

Suppose we have a Klein-4 group (G, $\bullet$) with elements are four different domino cards. Let us define the binary operation $\bullet$: *deleting the same pattern on card.* This binary operation will implies $\bullet$ only be able to operate between two cards which have at least one side the same pattern. For example

$$(6,1) \bullet (0,6) = (0,1)$$

$$(3,0) \bullet (0,2) = (2,3)$$

Following Klein-4 group definition, we need to have four different cards as elements including the identity.

**Theorem 2.1.** *Let (G,•) be a Klein-4 group with • define as "deleting the same pattern on card". Then (0, 0) is the identity.*

*Proof.* Let $(p, q)$ be an arbitrary card element of (G,•) such that $0 \leq p, q \leq 6$. Since we need to have a card which operated by • to it self produces identity, then $(p, q)$•$(p, q)$ will implies $p$ deleting $p$ and $q$ deleting $q$, hence we have $(p, q)$•$(p, q)$=$(0, 0)$. Then we obtain our assertion. □

**Remark 2.2.** *Remaining three elements of Klein-4 group are defined as the combination of (p, q) • (q, r) = (p, r), where* $0 \leq p, q, r \leq 6$.

As we stated $(0, 0)$ to be our identity, here we attach all combinations of possible elements of Klein-4 group on Domino card with binary operation •.

| $e$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| (0,0) | (0,1) | $(1,2)$ | $(0,2)$ |
| | | $(1,3)$ | $(0,3)$ |
| | | $(1,4)$ | $(0,4)$ |
| | | $(1,5)$ | $(0,5)$ |
| | | $(1,6)$ | $(0,6)$ |
| | (0,2) | $(2,3)$ | $(0,3)$ |
| | | $(2,4)$ | $(0,4)$ |
| | | $(2,5)$ | $(0,5)$ |
| | | $(2,6)$ | $(0,6)$ |
| | (0,3) | $(3,4)$ | $(0,4)$ |
| | | $(3,5)$ | $(0,5)$ |
| | | $(3,6)$ | $(0,6)$ |
| | (0,4) | $(4,5)$ | $(0,5)$ |
| | | $(4,6)$ | $(0,6)$ |
| | (0,5) | $(5,6)$ | $(0,6)$ |

Table 2: Combinations of all possible elements

**Theorem 2.3.** *Let (p,q) be a non-identity element of (G,•). Maximum order of (p,q) is 2.*

*Proof.* Since $(p, q)$ is a non-identity of (G,•), then value of both of $p$ and $q$ are between 0 and 6 where $p = 0$ and $q = 0$ will not exists on the same card. This condition leads us to $(p, q)^1 = (p, q)$ and $(p, q)^2 = (p, q) \bullet (p, q) = e$. Then we obtain our assertion. □

# References

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