

Quantitative Analysis of Magnetohydrodynamic Sustained Convective Flow via Vertical Plate

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Abstract

In this paper, the effects of heat and mass transfer on MHD flow in an incompressible, heated fluid that has been accelerated to a steady free stream are investigated. It examines mass movement in a magnetic field as well as heat absorption qualities. In the model, non-linear governing equations and the Laplace transform method are applied. The relationship between temperature, concentration, and velocity and flow parameters is illustrated by studies utilizing parametric data.

Keywords: Magnetohydrodynamics, Laplace Transform method, Free Convection, vertical Plate

1 Introduction

The growth of an electrically directed fluid in a magnetic field is studied by MHD, a fluid element. It can be used in many different ways, including sun-facing cookers, concentrators that use sunlight, and self-explanatory solar-powered gatherers. These appliances can be used for a variety of tasks, including expanding waste water disappearing rates, cooking, broiling, and refining. Laplace transform is an essential technique in domains such as quantum material science, basic design, electrical and electronic building, fluid physics, and basic design because it converts differential situations into elementary mathematical structures. Research progress has made it possible to recreate Laplace transformable circumstances.



The effects of thermo-dispersion, warm age/assimilation, and depicted movement were investigated by H.T. Kataria and Patel [1] on a second-grade fluid hazardous-free convective MHD stream that is artificially responsive and transmits an infinite vertical plate. The repercussions of unstable MHD free convection flow via a vertical permeable moving plate with radiation for heat and mass transport were studied by M.C. Krishna Reddy et al. [2]. The effects of an unstable hydromagnetic free convective flow across an infinite vertical plate submerged in a porous environment with heat absorption were examined by Murali et al. [3] in order to determine the mass and heat transfer consequences. Deepa Gadipally, et al. [4] used the finite difference method to analyze the effects of Soret and Dufour on unsteady MHD flow across a semi-infinite vertical porous plate, while Murali et al. [5] examined their influence on unstable hydromagnetic free convective fluid flow. For a system that solves problems using MHD, Murali et al. [6]–[7] offered a finite element solution. N.V.N. Babu et al. [8] conducted an analysis of the Casson fluid performance on natural convective dissipative Couette flow via an infinite vertically inclined plate. A numerical analysis of the convective MHD Jeffrey Fluid Flow resulting from vertical plate movements was carried out by Murali et al. [9]. Muthucumaraswamy and Geetha [10] talk about impacts of explanatory movement on an isothermal vertical plate with consistent mass transition.

Velocity, mass and temperature analysis of gravity-driven convection nanofluid flow past an oscillating vertical plate was studied by [11]–[12]. A few writers have examined the consequences of heat and mass transfer, and their analyses have greatly influenced our comprehension of the nature of the work that has been documented [13]–[21].

Heat absorption and temperature variations were taken into account when analyzing the MHD heat and mass transfer flow via vertical plate using the Laplace transform method. The method effectively understood coupled non-straight halfway differential conditions and deduced the local Nusselt number, neighboring Sherwood number, and adjacent skin-friction coefficients.

2 Material and Methods

An incompressible fluid's unstable MHD flow was examined using an infinite vertical plate that was uniformly heated, with variable mass diffusion, and exponential acceleration. At first, there is a concentration level at each site, and the fluid and plate are at the same temperature and motionless.

Fig. 1 depicts both the physical illustration and the coordinate arrangement. This inquiry assumes fluid physical attributes are stable, with a transverse magnetic field applied to the plate. The induced magnetic field is smaller due to low conductive quality. Viscous dissipation and Joule heating are ignored.

Boussinesq's approximation states that the following set of equations controls the unsteady flow:

Momentum Equation:

$$\left[\frac{\partial u'}{\partial t'} \right] = g \left[\frac{\partial^2 u'}{\partial y'^2} \right] - \left[\frac{\sigma B_o^2}{\rho} \right] u' + [g\beta(T' - T'_\infty)] + [g\beta^*(C' - C'_\infty)] \tag{1}$$

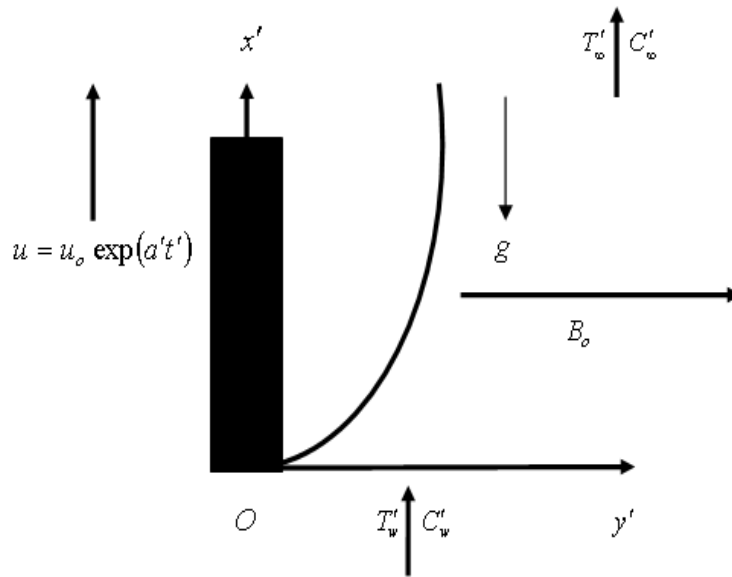


Figure 1. Physical formation and coordinate system

Energy Equation:

$$C_p \left[\frac{\partial T'}{\partial t'} \right] = \kappa \left[\frac{\partial^2 T'}{\partial y'^2} \right] + [Q_o(T'_\infty - T')] \quad (2)$$

Species Diffusion Equation:

$$\left[\frac{\partial C'}{\partial t'} \right] = D \left[\frac{\partial^2 C'}{\partial y'^2} \right] - D[K_r'(C' - C'_\infty)] \quad (3)$$

The corresponding initial and boundary conditions are

$$\left. \begin{aligned} t' \leq 0: & \quad u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y' \\ t' > 0: & \quad \left\{ \begin{aligned} u' = u_0 \exp(a't'), T' = T'_\infty + (T'_w - T'_\infty) At', C' = C'_\infty + (C'_w - C'_\infty) At' \text{ at } y' = 0 \\ u' = 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \end{aligned} \right\} \quad (4)$$

On introducing the following non-dimensional quantities into the Eqs. (1), (2) and

(3)

$$\left. \begin{aligned} u = \frac{u'}{u_0}, t = \frac{t'u_0^2}{g}, y = \frac{y'u_0}{g}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, M = \frac{\sigma B_0^2 g}{\rho u_0^2}, Gr = \frac{g\beta g(T'_w - T'_\infty)}{u_0^3}, \\ Gc = \frac{g\beta^* g(C'_w - C'_\infty)}{u_0^3}, Pr = \frac{gC_p}{\kappa}, Sc = \frac{g}{D}, S = \frac{Q_0 g^2}{\kappa u_0^2}, a = \frac{a' g}{u_0^2}, \lambda = \frac{gK_r'}{u_0^2}, Re_x = \frac{u_0 x}{g}, A = \frac{u_0^2}{g} \end{aligned} \right\} \quad (5)$$

Changed governing equations as follows:

Momentum Equation:

$$\frac{\partial u}{\partial t} = Gr\theta + Gc\phi + \frac{\partial^2 u}{\partial y^2} - Mu \quad (6)$$

Energy Equation:

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{S}{Pr} \theta \quad (7)$$

Concentration Equation:

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - \frac{\lambda}{Sc} \phi \quad (8)$$

Boundary conditions for the flow are as follows:

$$\begin{aligned}
 & t \leq 0: u = 0, \theta = 0, \phi = 0 \text{ for all } y \\
 & t > 0: \left\{ \begin{aligned} & u = \exp(at), \theta = t, \phi = t \text{ at } y = 0 \\ & u = 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\}
 \end{aligned}
 \tag{9}$$

The following are important parameters like Skin-friction, Nusselt number and Sherwood number

$$Cf = -\left(\frac{\tau'_w}{\rho u_o \mathcal{G}}\right) = -\left(\frac{\partial u}{\partial y}\right)_{y=0}
 \tag{10}$$

$$Nu = -x \frac{\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}}{T'_w - T'_\infty} \Rightarrow Nu Re_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}
 \tag{11}$$

$$Sh = -x \frac{\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}}{C'_w - C'_\infty} \Rightarrow Sh Re_x^{-1} = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}
 \tag{12}$$

3 Results and Discussions

The following equations were modeled for an unsteady MHD convective problem and solved using the laplace transform approach.

$$\theta(y,t) = \left(\frac{t}{2} + \frac{y \text{Pr}}{4\sqrt{S}}\right) \exp(y\sqrt{S}) \operatorname{erfc}\left(\frac{y\sqrt{\text{Pr}}}{2\sqrt{t}} + \sqrt{\frac{St}{\text{Pr}}}\right) + \left(\frac{t}{2} - \frac{y \text{Pr}}{2\sqrt{S}}\right) \exp(-y\sqrt{S}) \operatorname{erfc}\left(\frac{y\sqrt{\text{Pr}}}{2\sqrt{t}} - \sqrt{\frac{St}{\text{Pr}}}\right) \tag{13}$$

$$\phi(y,t) = \left(\frac{t}{2} + \frac{y}{4}\sqrt{\frac{Sc}{\mu}}\right) \exp(y\sqrt{\mu Sc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{\mu t}\right) + \left(\frac{t}{2} - \frac{y}{4}\sqrt{\frac{Sc}{\mu}}\right) \exp(-y\sqrt{\mu Sc}) \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{\mu t}\right) \tag{14}$$

$$u(y,t) = \frac{e^{\alpha}}{2} \left[e^{\sqrt{M+a}y} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(M+a)t}\right) + e^{-\sqrt{M+a}y} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(M+a)t}\right) \right] \\ - \frac{\alpha}{2} \left[\frac{e^{\alpha}}{b^2} \left\{ e^{\sqrt{M+b}y} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(M+b)t}\right) + e^{-\sqrt{M+b}y} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(M+b)t}\right) + \exp\left(y\sqrt{\text{Pr}\left(\frac{S}{\text{Pr}}+b\right)}\right) \right\} \right. \\ \left. + \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\text{Pr}}{t}} + \sqrt{\left(\frac{S}{\text{Pr}}+b\right)t}\right) - \exp\left(-y\sqrt{\text{Pr}\left(\frac{S}{\text{Pr}}+b\right)}\right) \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\text{Pr}}{t}} - \sqrt{\left(\frac{S}{\text{Pr}}+b\right)t}\right) \right] + \\ + \frac{1}{b} \left(t + \frac{1}{b} + \frac{y}{2\sqrt{M}} \right) \exp(y\sqrt{M}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{Mt}\right) - \frac{1}{b} \left(t + \frac{1}{b} - \frac{y}{2\sqrt{M}} \right) \exp(-y\sqrt{M}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{Mt}\right) \\ + \frac{1}{b} \left(t + \frac{1}{b} + \frac{y \text{Pr}}{2\sqrt{S}} \right) \exp(y\sqrt{S}) \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\text{Pr}}{t}} + \sqrt{\frac{St}{\text{Pr}}}\right) + \frac{1}{b} \left(t + \frac{1}{b} - \frac{y \text{Pr}}{2\sqrt{S}} \right) \exp(-y\sqrt{S}) \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\text{Pr}}{t}} - \sqrt{\frac{St}{\text{Pr}}}\right)$$

Using (10) and (15), Skin-friction coefficient is of the following form

$$C_f = e^{\alpha} \left[\sqrt{(M+a)} \left(1 - \operatorname{erfc}\left(\sqrt{(M+a)t}\right) \right) + \frac{1}{\sqrt{\pi t}} \exp\left(-\sqrt{(M+a)t}\right) \right] + \\ \alpha \left[\frac{\exp(bt)}{b^2} \left\{ \sqrt{(M+b)} \left(\operatorname{erfc}\left(\sqrt{(M+b)t}\right) - 1 \right) - \frac{1}{\sqrt{\pi t}} \exp\left(-\sqrt{(M+b)t}\right) \right\} \right. \\ \left. + \left[-\sqrt{\text{Pr}\left(\frac{S}{\text{Pr}}+b\right)} \left(\operatorname{erfc}\left(\sqrt{\left(\frac{S}{\text{Pr}}+b\right)t}\right) - 1 \right) + \sqrt{\frac{\text{Pr}}{\pi t}} \exp\left(-\sqrt{\left(\frac{S}{\text{Pr}}+b\right)t}\right) \right] \right] + \\ - \frac{1}{2b} \left(\frac{\text{Pr}}{\sqrt{S}} \right) \left(1 - \operatorname{erfc}\left(\sqrt{\frac{St}{\text{Pr}}}\right) \right) - \frac{1}{b} \left(t + \frac{1}{b} \right) \left\{ \sqrt{S} \left(1 - \operatorname{erfc}\left(\sqrt{\frac{St}{\text{Pr}}}\right) \right) + \left(\sqrt{\frac{\text{Pr}}{\pi t}} \right) \exp\left(-\sqrt{\frac{St}{\text{Pr}}}\right) \right\} \\ - \psi \left[\frac{\exp(dt)}{d^2} \left\{ \sqrt{(M+d)} \left(\operatorname{erfc}\left(\sqrt{(M+d)t}\right) - 1 \right) - \frac{1}{\sqrt{\pi t}} \left(\exp\left(-\sqrt{(M+d)t}\right) - \sqrt{Sc(\mu+d)} \left(\operatorname{erfc}\left(\sqrt{(\mu+d)t}\right) - 1 \right) \right) \right\} \right. \\ \left. + \left(\sqrt{\frac{Sc}{\pi t}} \right) \exp\left(-\sqrt{(\mu+d)t}\right) \right] \\ + \frac{1}{2b\sqrt{M}} \left(1 - \operatorname{erfc}\left(\sqrt{Mt}\right) \right) + \frac{1}{d} \left(t + \frac{1}{d} \right) \left\{ \sqrt{M} \left(1 - \operatorname{erfc}\left(\sqrt{Mt}\right) \right) + \frac{1}{\sqrt{\pi t}} \left(\exp\left(-\sqrt{Mt}\right) \right) \right\} - \frac{1}{2d} \sqrt{\frac{Sc}{\mu}} \left(1 - \operatorname{erfc}\left(\sqrt{\mu t}\right) \right) \\ - \frac{1}{d} \left(t + \frac{1}{d} \right) \left\{ \sqrt{\mu Sc} \left(1 - \operatorname{erfc}\left(\sqrt{\mu t}\right) \right) + \sqrt{\frac{Sc}{\pi t}} \left(\exp\left(-\sqrt{\mu t}\right) \right) \right\} \tag{16}$$

4 Conclusions

It can be concluded that life expectancy in various provinces in Indonesia has increased consistently from 2019 to 2024. Although there are variations between provinces, data that frequently appears shows a significant increase in population life expectancy, reflecting improvements in overall health and quality of life. Based on the calculation results, it shows that women tend to have a higher life expectancy than men in each province, and this gender difference is relatively stable from year to year. The average prediction of AHH based on gender in Indonesia shows an increase and decrease. This increase and decrease occur in every data used and also in every calculation tool used.

It can be concluded that in terms of the calculation tool used, the Lagrange polynomial interpolation method can be implemented to predict life expectancy in Indonesia because it has a Root Mean Squared Error with a value of 0.085875 or around 8.58%, which means it is included in the moderate category, which means the prediction results are included in the category which can be good [8]. And implementing Lagrange polynomial interpolation on an application in the form of a website can make it easier to carry out calculations automatically.

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