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Numerical Beamforming and Parametric Descriptions of Laguerre-Gaussian Vortex Beams

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Abstract

Vortex beams are beams with a helical wavefront that have found applications in optical or acoustic tweezers to manipulate microscopic particles. Vortex beam imposes torque or force to particles, allowing them to trap the object within the beam's field and induce motion or displacement in a non-contact manner. One type of such beam is a Laguerre-Gaussian beam, where the solution of a Gaussian wave is modified by the Laguerre polynomial term that determines the pattern and helical characteristic of the beam. In this paper, a numerical method based on the mathematical expression of the Laguerre-Gaussian beam is implemented to describe how the parameters change the physical behavior of the beam. This work has shown that a straightforward numerical method is capable of producing this kind of beam. Therefore, this approach can be used for generating vortex beams for physical emissions, complex numerical simulations, or demonstrations for teaching purposes.

Keywords: vortex beam, laguerre-gaussian beam, numerical beamforming

1 Introduction

Vortex beam, characterized by the twisting of its phase as it propagates, has been investigated for its potential in the engineering field. This helical type of beam may prevail in the acoustical and optical form. One of the attractive properties of this kind of wave is that it carries angular momentum, which in turn can be transferred into the medium. Therefore, it is possible to take advantage of this feature to introduce motions to the particles within the beam envelope, creating optical or acoustic tweezers which not only traps particles but also manipulates their motion and position in a contactless manner. Practically, the tweezers can be implemented to hold position and control

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orientation of molecules or cells for microscopy [1]. It has also been demonstrated that a tweezer is capable of stretching microparticles to evaluate their viscoelastic property [2,3]. Both acoustic and optical tweezers have already been implemented in various research areas involving micro-sized objects such as in the biomedical and nanomaterial fields.

Multiple methods have been developed to generate vortex beams. In acoustic beamforming, a common method involves an array of piezoelectric transducers by modulating the phase and amplitude of each transducer within the array in order to form the beam [4]. Recently, Zhou et. al. proposed another method using a spiral diffraction grating and worked out theoretical and numerical investigations to reveal its feasibility [5]. On the other hand, various methods also exist to generate optical vortex, some of which are by employing laser with diffraction grating [6], spatial light modulator [7], and deformable mirror [8]. With various beam forming methods available and being developed, research on the vortex beam will advance to broaden its application in the industrial or scientific domains.

The aim of this paper is to demonstrate the principal properties of the acoustic vortex beam by visualizing the shape of the beam as it propagates along the medium and to describe theoretically the role of parameters in the mathematical expression of the Laguerre-Gaussian vortex beam. This paper isolates the study solely on the nature of the propagation of the ideal beam in a lossless medium. Physical phenomena related to the interaction of waves with the medium such as viscosity effects, damping and other external forces are not considered in this paper.

Vortex beams studied in this work are formed as Laguerre-Gaussian beams. It is a Gaussian function with Laguerre polynomial term which determines the radial and azimuthal pattern of the beam.

2 Research Methodology

Mathematical expression that describes the solution of a Laguerre-Gaussian beam in cylindrical coordinates as a function of the radius (r), azimuthal angle (φ) and propagation distance (z) can be described by [9]:

$$u(r,\phi,z) = A_0\left(\sqrt{2}\frac{r}{w(z)}\right)^l L_p^l\left(2\frac{r^2}{w(z)^2}\right)\frac{w_0}{w(z)}\ exp[-i\varphi_{pl}(z)]\ exp\left[i\frac{k}{2q(z)}r^2\right]\ exp(il\phi) \quad (1)$$

where A_0 is a constant and L_p^l is the associated Laguerre polynomial of the argument that follows, i.e. the $\left(2\frac{r^2}{w(z)^2}\right)$. The associated Laguerre polynomials $L_p^l(x)$ satisfy the following differential equation:

$$x\frac{d^2L_p^l}{dx^2} - (l+1-x)\frac{dL_p^l}{dx} + pL_p^l = 0 \quad (2)$$

The beam waist and the beam width at the distance z is denoted by w_0 and $w(z)$, respectively. Gouy phase shift $\varphi_{pl}(z)$ is given by:

$$\varphi_{pl}(z) = (2p+l+1)tan^{-1}(z/z_0) \quad (3)$$

In the case of parameter p and l both equal to zero, a fundamental Gaussian beam will appear. The Rayleigh length z_0, which is the distance from the beam focus where the beam area doubles, can be found by knowing the wavelength (λ) or wavenumber (k) through:

$$z_0 = \frac{\pi {w_0}^2}{\lambda} = \frac{k{w_0}^2}{2} \quad (4)$$

The Laguerre-Gaussian beam is modeled in MATLAB 2020b by developing a code to solve the mathematical expression as discussed above. An advantage of using MATLAB 2020b is that the associated Laguerre polynomial term can be simply obtained by a built-in function *laguerreL(p,l,x)* [10]. Here, the p and l inputs are set as integers to represent the modes and the x input is fed by the $\left(2\frac{r^2}{w(z)^2}\right)$ term. The program loop iterates in the increment of radial and angular positions to determine the solution at each node throughout the specified area of square with the sides of 4λ. That iteration can then be repeated at different distance in front of the source plane (planes at $z > 0$) to reveal the angular displacement of intensity peaks.

This study attempts to examine the effect of varying physical parameters that can be adjusted by the setting of the wave source or transducer, namely the frequency (f). The excited frequency determines the beam's wavelength (λ) depending on the speed of sound

(c) as an intrinsic property of the medium of propagation, via the well-known relation of $\lambda = c/f$.

3 Results and Discussions

Different modes of LG beams can be described by the parameter p and l in the $L_p^l(x)$ associated Laguerre polynomial. There, p and l represent the radial (center to periphery direction) and azimuthal (angular-wise direction) variation of the beam, respectively. Several examples are presented in Figure 1, depicting the amplitude field in the beam source plane ($z = 0$) for p and l from 0 to 4. It can be observed that the radial pattern of the beam, which is simply the repetition of each beam's azimuthal pattern towards the radial direction, is described by the parameter p (the degree of the associated Laguerre polynomial). Meanwhile, the azimuthal pattern itself is related to the integer l. For brevity, the mode will be further designated as LG_{pl}. An LG_{00} mode has zeroes on both p and l which results in a standard Gaussian beam, having a single peak at the center. The other LG modes with one or both non-zero p and l values have multiple peaks in azimuthal and/or radial direction. The regularity extends in the same fashion for any integer values of p and l.

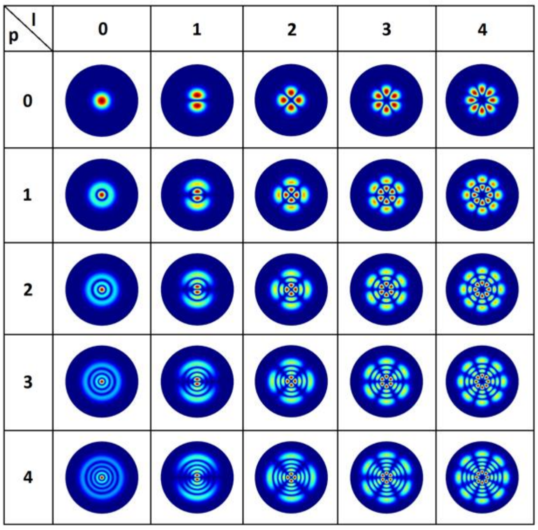

Figure 1. Amplitude pattern of various modes of LG beam at the source plane ($z = 0$), described by the p and l parameter of the associated Laguerre polynomials.

On the plane of the beam source (at $z = 0$), the amplitude distribution of each mode is described by the p and l parameters of the associated Laguerre polynomials. Varying p and l results in the donut or segmented shapes with different numbers of layers of amplitude peaks in the radial direction. However, the total amplitude across the surface is independent of the LG modes if the other variables remain constant, as expected from the expression in Equation 1. Accordingly, the mode only specifies how the amplitude distributes within the plane orthogonal to the propagation direction.

The amplitude patterns as shown in Figure 1 consist of multiple lobes or peaks, with an exception for the LG_{00} mode that has only central peak following a standard Gaussian distribution. Note that the amplitude is plotted as absolute value, hence the maxima can be either positive or negative with the main purpose of indicating the pattern of intensity strength relative to zero, neglecting the phase evolution. The phase-dependent shape, however, can be investigated in Figure 2, where actually at a particular time, the peak may be directed towards the positive or negative sign. Take an example of mode LG_{02} that corresponds to 2D image with $p = 0$ and $l = 2$ in Figure 1 and 3D plot in Figure 2(d). In Figure 1, LG_{02} shows 4 symmetric lobes meanwhile in reality the 4 peaks consist of 2 lobes in positive direction and other 2 in negative.

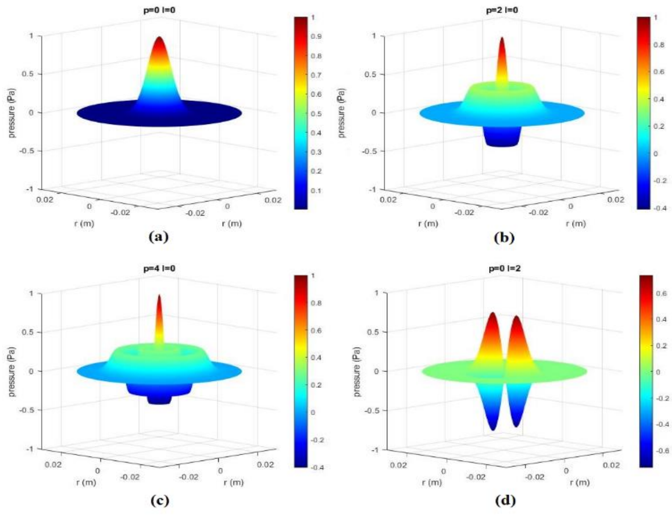

Figure 2. Implementation on acoustic beam: pressure amplitude profile at a given phase for (a) LG_{00}; (b) LG_{20}; (c) LG_{30}; (d) LG_{22}

A case study is carried out in acoustic setup by implementing the numerical method, where snapshots of pressure distribution that occurs at a certain phase for select

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modes are shown in Figure 2. Several instances of LG modes: LG_{00}, LG_{20}, LG_{40} and LG_{02} are presented. The beam in this case is simulated as an acoustic perturbation at the frequency of 1 MHz with the velocity of 1490 m/s, resembling sound propagation in water, hence the wavelength is 1.49 mm. Beam waist is set as 4 times wavelength, or equals to 5.96 mm in this case. The simulation is limited to observe the area within the radius of 20 times the wavelength or equals to 29.8 mm radially outward from the beam's axis. That envelope is considered sufficient to capture far field region without exaggerating computational requirements. The figures reveal that in a higher polynomial degree of LG mode, for example LG_{20} and LG_{40}, there exist valleys and peaks which appear in an alternating fashion along the radial direction. In addition, the same behavior of alternating peaks and valleys also prevails in the azimuthal direction. Taking the LG_{02} mode as an example, the peaks and valleys take place alternatingly every 90^{o} azimuthal angle, as demonstrated in Figure 2(d) Without the need to elaborate the other examples, this alternating patterns in both radial and azimuthal direction persist for other modes as well.

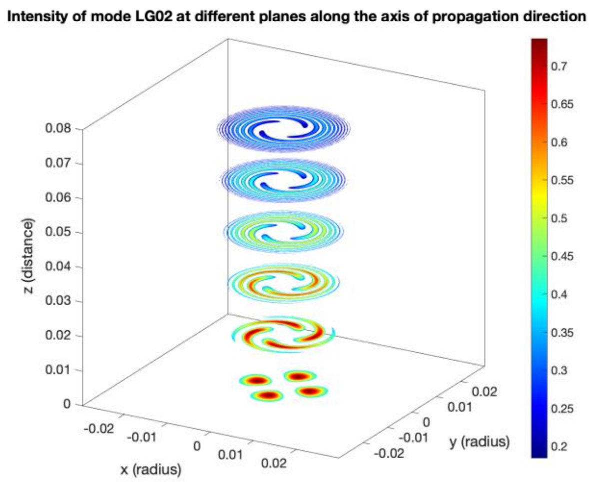

Figure 3. Amplitude field of LG_{02} on different planes at different positions downstream the source, showing spirals that indicates twisting behavior of the beam.

The use of different frequencies or mediums, which means different wave velocity, does not affect the shape of amplitude distribution at the beam source plane. That being said, all the preceding results and visualizations in Figure 1 and Figure 2 regarding the in-plane pressure distribution at $z=0$ remain identical regardless of the

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frequency and wave velocity settings, provided the same diameter of beam waist w_0 is maintained.

Rotational behavior of the Laguerre-Gaussian beam is observable down the propagation path, as presented in Figure 3. The amplitude fields on planes at different *z*-position form spiral shapes which indicates angular displacement of the beam as the amplitude peak moves in azimuthal direction. The field radius expands as the waves propagate further from the source plane, and at the same time the overall amplitude decreases as indicated by the corresponding color scale.

4 Conclusions

This work has demonstrated a numerical method to visualize a Laguerre-Gaussian beam. The physical implication of each parameter and terms in the mathematical expression has been discussed, where the radial and azimuthal patterns are determined by the Laguerre polynomial parameters. On the other hand, other wave parameters such as frequency, wavelength, beam width and amplitude do not affect the peak distribution and angular displacement pattern. The vortex behavior has also been visualized by depicting the field on planes at different positions ahead of the beam source which shows spiral shapes.

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