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# Enhancing the Cooling Effectiveness Utilizing a Tapered Fin Having Capsule-Shaped Cross-Sectional Area Numerically Simulated Using Finite Difference Method

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**Abstract**

This paper reports the results of our research on improving the cooling of an engine using fins. This problem is important to discuss because various parts of the world have utilized machining technology. When the engine operates, it produces heat. This heat reduces the efficiency of the engine's performance. In this problem, we developed a tapered fin method with a capsule cross-section to enhance cooling performance. The fin consists of two different materials that are perfectly joined. In this paper, the fin analysis is performed using the explicit finite difference numerical method. This method simulates the heat distribution on the fins. The results of our research include temperature distribution, heat flow rate, efficiency, and fin effectiveness in unsteady-state conditions with variations in material composition. The highest heat flow rate, fin efficiency, and fin effectiveness were achieved with a fin material composition of copper and aluminum, yielding an efficiency value of 0.89 and an effectiveness of 20.7. Our research results offer potential for the industry to design fins for innovative applications.

**Keywords**: Effectiveness, Efficiency, Fin, Finite Difference

# 1 Introduction

Recently, energy use is increasing in a more efficient, flexible, sustainable manner. Several innovative techniques and applications continue to be developed, such as small modular reactors (SMR) [1], [2], such as power generation systems with advanced operating parameters (high operating temperatures) [3]. In implementing the above innovation, a heat exchanger is required that possesses high efficiency and effectiveness. The ideal heat exchanger for this application must have a large heat transfer area while considering the narrow construction [4]. The one common heat exchanger option to increase the heat transfer rate is fins[5]. The fins serve to increase the area [6]. Examples of fin applications that we often encounter include computer processors, combustion chambers in combustion engines, electronic equipment, radiators, and other heat exchangers [7]. The larger number of fins installed, the greater the amount of heat released by those fins [8].

The efficiency and effectiveness of the fin are important considerations in fin design. The efficiency and effectiveness of fins can be investigated using analytical, experimental, and computational methods. In previous research, the one-dimensional case of a straight rhombus cross-section fin was analyzed using the explicit finite difference method [7]. In other studies, various cross-sectional shapes of fins, such as circular, pentagonal, and rectangular, have also been investigated [9], [10], [11], [12]. In this study, the efficiency and effectiveness of the fins were simulated using the explicit finite difference method. From various previous studies, no research has reported findings on the efficiency and effectiveness of tapered fins with a capsule-shaped cross-section composed of two different materials.

This paper aims to research temperature distribution in tapered fins that have a capsule cross-section consisting of two materials under unsteady conditions. This research uses the explicit finite difference method. The explicit finite difference method provides a fast solution, and its accuracy can be adjusted by modifying the size and number of nodes [13]. The results obtained from the temperature calculation equation in each node point were used to develop a computational program. In addition to calculating the temperature distribution, the program is also designed to calculate the heat flow rate, efficiency, and fin effectiveness, as well as to observe the effect of changes in one of the fin materials on distribution, heat flow rate, efficiency, and fin effectiveness.

# 2 Material and Methods

This study employs a computational approach based on the explicit finite difference method. Figure 1 shows an image of the tapered straight fin under investigation. The tapered fin has a capsule-shaped cross-section, with a varying cross-sectional area along its length, and is composed of two different materials. The height of the fin base capsule is D = 0.01 m, and its width is D + a, where value a = 0.03 m. The height of the fin tip capsule is denoted as d = 0.005 m, and its width is d + a. The total length of the straight fin is L = 0.1 m. The length of the fin made of material 1 is denoted by $L_1$, and the length of the fin made of material 2 is denoted by $L_2$ The fin is divided into 25 small elements, known as nodes, each spaced equally by a distance of $\Delta x$. The initial temperature of the fin is uniform and equal to the base temperature, i.e., $T_i = T_b$ =100°C. The fin is then exposed to a fluid environment with a constant and uniform temperature $T_\infty$ = 30°C. The convection heat transfer coefficient is assumed to be constant, with a value of $h = 50$ $W/m^2$°C. The base temperature is maintained constant over time. The entire lateral surface of the fin and the fin tip are in contact with the surrounding fluid. Heat conduction is assumed to take place solely in the $x$-direction (one direction). The properties of the fin

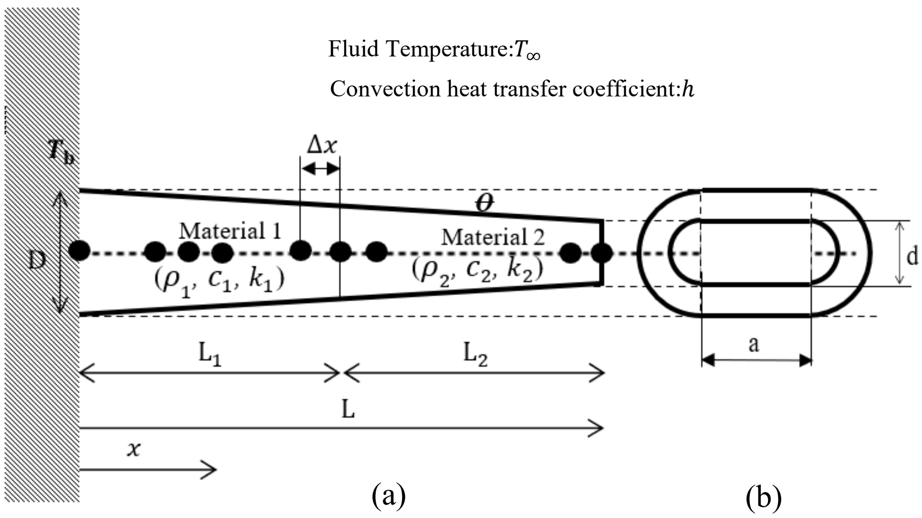


**Figure 1**. Tapered fin with capsule cross-section, (a) side view, (b) front view

material (density $\rho$, specific heat capacity $c$, and thermal conductivity $k$) were assumed to be constant and independent of temperature changes.

The mathematical model used to calculate the temperature at position x on the fin and at time $t$ is a partial differential equation. The differential equation for this case is expressed in Equation (1):

$$\frac{\partial^2 T(x,t)}{\partial x^2} + \frac{hAs}{kA_p}(T(x,t) - T_\infty) = \frac{1}{\propto}\frac{\partial T(x,t)}{\partial t}, \qquad 0 < x < L, t > 0 \tag{1}$$

$$\alpha = \frac{k}{\rho c}, \tag{2}$$

and boundary condition:

$$T(0,t) = T_b, \qquad x = 0, t > 0. \tag{3}$$

The formulation of transient heat conduction problems differs from that of steady-state conduction problems because transient cases involve an additional term representing the change in the energy content of the medium over time. This additional term appears as the first derivative of temperature with respect to time in the differential equation and as the change in internal energy content over a time interval $\Delta t$ in the energy balance formulation [14]. The energy balance on a volume element during a time interval $\Delta t$ (without energy generation) can be expressed as [14] :

$$\begin{pmatrix}\text{The total amount of energy}\\ \text{that flows into the volume element}\\ \text{over the time period } \Delta\text{t}\end{pmatrix} = \begin{pmatrix}\text{The change in the energy}\\ \text{content of the volume}\\ \text{element during } \Delta\text{t}\end{pmatrix}$$

It can be shown in Equation 4:

$$\sum_{i=1}^{k} q_i^n = \rho_i V_i c_1 \frac{\Delta T}{\Delta t}. \tag{4}$$

Based on Equation (4), the temperature within the volume elements between the base and the tip of the fin (excluding the interface volume element between the two fin materials) can be determined using Equation (5). The temperature of the interface volume element between the two fin materials is determined using Equation (6), while the temperature at the fin tip volume element is determined using Equation (7).

$$T_i^{n+1} = \frac{\Delta t. k}{\rho. Vi. C. \Delta x} . \left[ Ap_{i-\frac{1}{2}}. (T_{i-1}^n - T_i^n) + Ap_{i+\frac{1}{2}}. (T_{i+1}^n - T_i^n) + Bi. As_{i.} (T_\infty - T_i^n) \right] + T_i^n, \tag{5}$$

$$T_i^{n+1} = \frac{\Delta t}{(\rho_1\, x\, c_1\, x\, V_1 + \rho_2\, x\, c_2\, x\, V_2)} \left[ k_1. Ap_{i-\frac{1}{2}}. \left( \frac{T_{i-1}^n - T_i^n}{\Delta x} \right) + k_2. Ap_{i+\frac{1}{2}}. \left( \frac{T_{i+1}^n - T_i^n}{\Delta x} \right) + h. As_i (T_\infty - T_i^n\ ) \right] + T_i^n, \tag{6}$$

$$T_i^{n+1} = \frac{\Delta t. k_2}{\rho_2. Vi. c_2. \Delta x} . \left[ Ap_{i-\frac{1}{2}}. (T_{i-1}^n - T_i^n) + Bi_2. Ap_{i.} (T_\infty - T_i^n) + Bi_2. As_i. (T_\infty - T_i^n) \right] + T_i^n. \tag{7}$$

Under unsteady-state conditions, the actual heat transfer rate of the fin ($q_{\text{actual}}^n$), the maximum heat transfer rate ($q_{\text{ideal}}^{\text{n}}$), and the heat transfer rate without the fin ($q_{\text{finless}}^n$), can be calculated sequentially using Equations (8), (9), and (10).

$$q_{\text{actual}}^n = h \sum_{i=1}^{m} \big( A_i (T_i^n - T_\infty) \big), \tag{8}$$

$$q_{\text{ideal}}^n = h \sum_{i=1}^{m} \big( A_i (T_b - T_\infty) \big), \tag{9}$$

$$q_{\text{finless}}^n = h\, A_d (T_b - T_\infty). \tag{10}$$

Under transient conditions, the efficiency of the fin can be determined using Equation (11):

$$\eta_{\text{fin}}^n = \frac{q_{\text{actual}}^n}{q_{\text{ideal}}^n} = \frac{h \sum_{i=1}^{m} \big( A_i (T_i^n - T_\infty) \big)}{h \sum_{i=1}^{m} \big( A_i (T_b - T_\infty) \big)}. \tag{11}$$

Under transient conditions, the fin effectiveness of the fin can be determined using Equation (12):

$$\varepsilon_{\text{fin}}^n = \frac{q_{\text{actual}}^n}{q_{\text{finless}}^n} = \frac{h \sum_{i=1}^{m} \big( A_i (T_i^n - T_\infty) \big)}{h\, A_d (T_b - T_\infty)}. \tag{12}$$

Table 1. Properties of the material [14]

| Material | Density ($\rho$) (kg/m$^3$) | Thermal Conductivity ($k$) (W/m$^2$°C) | Specific Heat ($c$) (J/kg°C) |
|---|---|---|---|
| Copper (Cu) | 8933 | 401 | 385 |
| Aluminum (Al) | 2702 | 237 | 903 |
| Zinc (Zn) | 7140 | 116 | 389 |

| | | | |
|---|---|---|---|
| Iron (Fe) | 7870 | 80.2 | 447 |
| Nikel (Ni) | 8900 | 444 | 90,7 |
| Bras (Cu-Zn) | 8933 | 110 | 380 |

# 3 Results and Discussions

The results of the temperature distribution, actual heat flow rate, fin efficiency, and fin effectiveness calculations are presented in Figures 2, 3, 4, and 5. Under unsteady-state conditions, the temperature distribution continues to change over time. In this study, since the base temperature $T_b$ is maintained at a constant 100°C and the fin is continuously exposed to the surrounding fluid, the temperature at each volume element continues to decrease until a steady-state condition is reached throughout the entire domain. In Figures 2 through 5, the temperature distribution is evaluated at 600 seconds by comparing the temperature at each volume element for various fin material compositions. The heat transfer rate and fin efficiency are evaluated at each time step for different material compositions

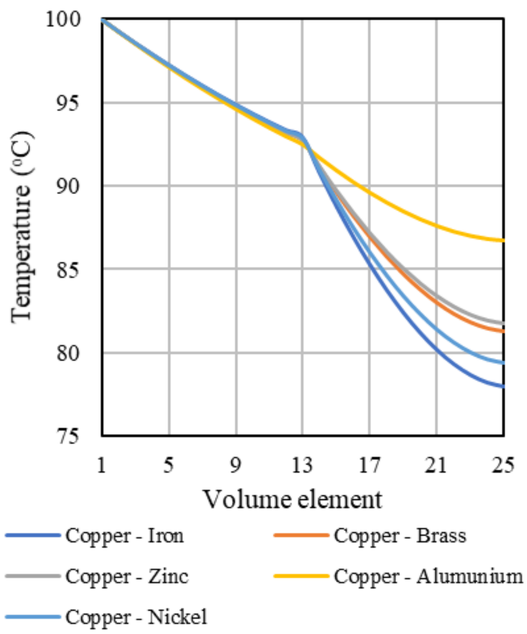


**Figure 2**. Fin temperature at t=600 s

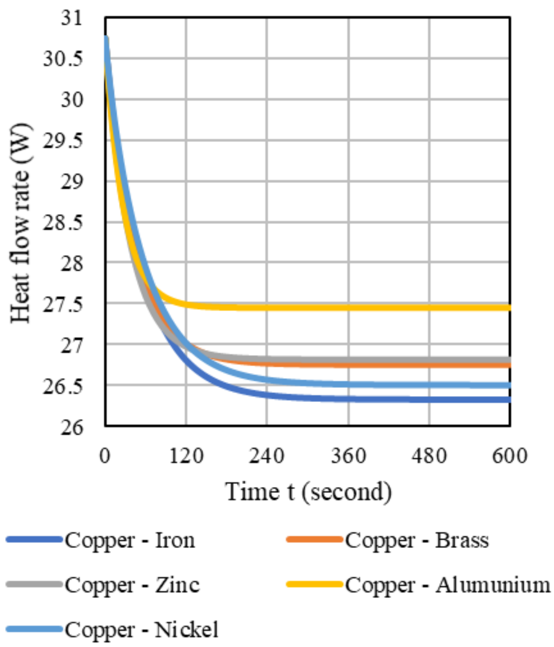


**Figure 3**. Heat transfer rate.

Figure 2 presents the temperature distribution along the fin for various material composition variations. At 600 seconds, the fin composed of copper–iron exhibits the greatest temperature drop, reaching 78°C from its initial condition. In contrast, the lowest temperature drop is observed in the copper–aluminum composition, with a final temperature of 86°C. The temperature distribution in the unsteady state is influenced by material properties such as density, thermal conductivity, and specific heat capacity, as shown in Table 1. This is because iron has a low thermal conductivity, resulting in less optimal heat transfer from the first material compared to other materials.

Figures 3, 4, and 5 present the actual heat flow rate from the fin, fin efficiency, and fin effectiveness. These calculated results all depend on the temperature distribution along the fin. The heat flow rate released by the fins, in sequence from the largest to the smallest is owned by the fin of the combinations of copper–aluminum, copper–zinc, copper–brass, copper–nickel, and copper–iron. This trend corresponds to the temperature distribution generated by each fin material combination. The higher the temperature distribution along the fin, the greater the heat flow rate. This is because a higher temperature along the fin results in a larger temperature difference with the surrounding fluid. Consequently, a

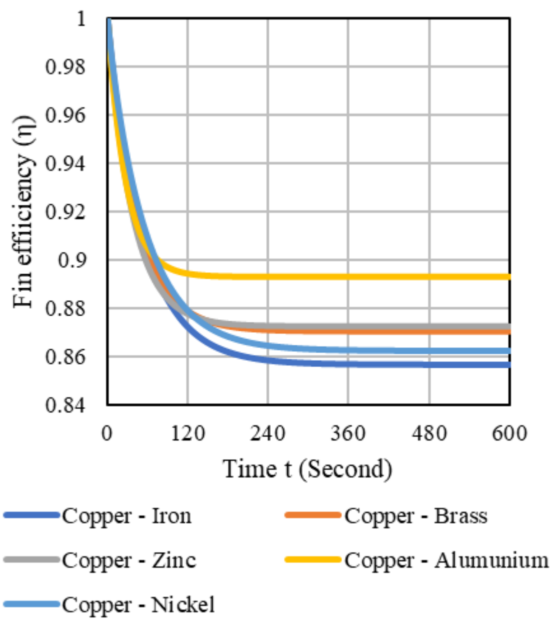


**Figure 4**. Fin efficiency

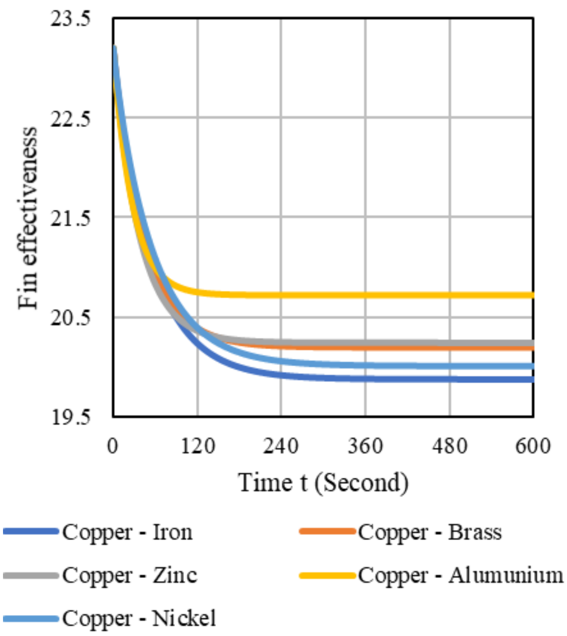


**Figure 5**. Fin effectiveness.

greater temperature difference leads to a higher rate of heat transfer. The highest efficiency and effectiveness are achieved by the copper–aluminum fin material composition, with an efficiency value of 0.89 and an effectiveness of 20.7.

# 4 Conclusions

Based on the research results, under unsteady-state conditions, the material properties that influence temperature distribution, heat flow rate, efficiency, and effectiveness are density, specific heat capacity, and thermal conductivity. The highest efficiency and effectiveness are achieved by the copper–aluminum fin composition, as it has the highest thermal diffusivity value. Our study is limited to several material compositions, convective heat transfer coefficients, initial temperatures, and fin geometries. Heat transfer is also assumed to occur only in a one-dimensional (1D) domain. Future research may expand this work by exploring a broader range of fin material compositions, two- or three-dimensional (2D/3D) fin problems, or materials with temperature-dependent properties.

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Nomenclature

| | | |
|---|---|---|
| $\Delta x$ | : | Distance between volumes element (m) |
| $\Delta t$ | : | Time step, from the $n$ to the $n+1$ iteration (s) |
| $T_\infty$ | : | Fluid temperature around the fin (°C) |
| $T_b$ | : | Temperature at the base of the fin, (°C) |
| $T_i^n$ | : | The temperature at the $i$ position, in the $n+1$ iteration (°C) |
| $T_i^{n+1}$ | : | The temperature at the $i$ position, in the $n+1$ iteration (°C) |
| $T_{i-1}^n$ | : | The temperature at the $i-1$ position, in the n iteration (°C) |
| $T_{i+1}^n$ | : | The temperature at the $i+1$ position, in the n iteration (°C) |
| $L_1$ | : | The length of the fin with material 1, m |
| $L_2$ | : | The length of the fin with material 2, m |
| L | : | The length of the fin, $L = L_1+L_2$, m |
| $A_p$ | : | Area of the cross section of the fin ($m^2$) |
| $As_i$ | : | Area of the surface of the volume element at the $i$ position touching the fluids around the fin ($m^2$) |

| | | |
|---|---|---|
| $k$ | : | Thermal conductivity $(\mathrm{W/m°C})$ |
| $k_1$ | : | Thermal conductivity of fin material 1 $(\mathrm{W/m°C})$ |
| $k_2$ | : | Thermal conductivity of fin material 2 $(\mathrm{W/m°C})$ |
| $h$ | : | Convection heat transfer coefficient $(\mathrm{W/m^2°C})$ |
| $\rho$ | : | The density of the fin material $(\mathrm{kg/m^3})$ |
| $\rho_1$ | : | The density of the fin material 1 $(\mathrm{kg/m^3})$ |
| $\rho_2$ | : | The density of the fin material 2 $(\mathrm{kg/m^3})$ |
| $c$ | : | Specific heat of the material, $(\mathrm{kJ/kg°C})$ |
| $c_1$ | : | Specific heat of the material 1, $(\mathrm{kJ/kg°C})$ |
| $c_2$ | : | Specific heat of the material 2, $(\mathrm{kJ/kg°C})$ |
| $\alpha$ | : | Thermal diffusivity of the material, $(\mathrm{m^2/s})$ |
| $Bi$ | : | Biot number |
| $Bi_1$ | : | Biot number of material 1 |
| $Bi_2$ | : | Biot number of material 2 |

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